

# Structure of Certain Nonnormal Operators II

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In this note we continue the study of certain nonnormal operators begun in [1]. As in [1] we use  $C^*$ -algebra methods to obtain information about single operators. The results of [1] are extended and in particular we give a decomposition and structure theory for nearly normal, nearly binormal and nearly quasinormal operators. Finally we present a new result about postliminal operators. Throughout  $H$  denotes a complex not necessarily separable Hilbert space. Let  $A$  be a bounded linear operator on  $H$ . We obtain information about  $A$  by studying the  $C^*$ -algebra  $C^*(A)$ , which is generated by  $A$  and 1. Our major tool in this investigation is the algebra  $\mathfrak{K}(H)$  of all compact linear operators on  $H$ . If  $S$  is a subset of  $\mathfrak{B}(H)$ , the algebra of all bounded linear operators on  $H$ , the commutant of  $S$  will be denoted by  $S'$ . For  $A, B \in \mathfrak{B}(H)$  we use  $A \leftrightarrow B$  and  $A \rightleftharpoons B$  for  $AB - BA = 0$  and  $AB - BA \in \mathfrak{K}(H)$ . The real (imaginary) part of  $A \in \mathfrak{B}(H)$  is denoted by  $\operatorname{Re} A$  ( $\operatorname{Im} A$ ). Among the properties of operators are some which are invariant under  $C^*$ -homomorphisms. Those we call  $C^*$ -properties. Examples are:  $A$  is normal, hyponormal or a contraction. If an operator  $A$  has a  $C^*$ -property modulo  $\mathfrak{K}(H)$ , we say it has nearly this property. Thus we speak of nearly normal or nearly quasinormal operators. Our basic reference for the theory of  $C^*$ -algebras will be the book by J. Dixmier [5].

The following lemma generalizes a result of Kaplansky [5, 4.7.20] and Lemma 4 of [1]. It is the key for the remainder.

**Lemma 1.** *Let  $\mathfrak{A}$  be a  $C^*$ -algebra of operators on  $H$  with  $1 \in \mathfrak{A}$ . Then there exists an index set  $J$  and a family of orthogonal projections  $\{P_0, P_\alpha\}_{\alpha \in J}$  with*

- (1)  $P_0 + \sum P_\alpha = 1$ ,  $P_0, P_\alpha \in \mathfrak{A}'' \cap \mathfrak{A}'$  for all  $\alpha \in J$ ,
- (2)  $P_0\mathfrak{A} \cap \mathfrak{K}(H) = (0)$  and
- (3)  $P_\alpha H = K_\alpha \otimes \mathbb{C}^{n_\alpha}$ ,  $P_\alpha\mathfrak{A} \cap \mathfrak{K}(H) = \mathfrak{K}(K_\alpha) \otimes 1$  and  $n_\alpha < \infty$ .

The  $P_\alpha$ , with  $\alpha \neq 0$ , are minimal central projections of  $\mathfrak{A}''$  and each algebra  $P_\alpha\mathfrak{A}''$ , with  $\alpha \neq 0$ , is a factor of type I with finite multiplicity  $n_\alpha$ .  $\mathfrak{A}''$  is of type I if  $P_0\mathfrak{A}$  is of type I. If  $\mathfrak{A}$  is separable  $J$  is countable and  $(1 - P_0)H$  is separable.