

# *The Converse to the Fixed Point Theorem of P. A. Smith: II*

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An action of the cyclic multiplicative group  $\mathbf{Z}_n$  on the manifold  $M$  is semi-free if either  $r(t, x) = x, \forall t \in \mathbf{Z}_n$ , or  $r(t, x) = x$  only if  $t = 1$ , holds for each point  $x$  in  $M$ . Two semi-free actions  $r_i : \mathbf{Z}_n \times M \rightarrow M$  ( $i = 0, 1$ ) are equivalent if there is a semi-free action

$$r : \mathbf{Z}_n \times (M \times [0, 1], M \times \{0 \cup 1\}) \rightarrow (M \times [0, 1], M \times \{0 \cup 1\})$$

so that  $r|_{M \times i}$  is equal to  $r_i$  ( $i = 0, 1$ ).

In this paper we shall always assume that  $M$ , semi-free group actions on  $M$ , and equivalences of group actions on  $M$  are all differentiable. Under these restrictions the fixed point set,  $F_r$ , of any semi-free action  $r : \mathbf{Z}_n \times M \rightarrow M$  is a differentiable submanifold of  $M$ . Furthermore the action of  $\mathbf{Z}_n$  is linear in some neighborhood of  $F_r$  in  $M$  in the following sense:

(1) There is a  $\mathbf{Z}_n$ -equivariant, linear bundle,  $\tau$ , normal to  $F_r$  in  $M$ , such that the  $\mathbf{Z}_n$ -action restricts on each fiber of  $\tau$  to a linear automorphism.

(2) If  $n = 2$ , this restriction to each fiber of  $\tau$  is just the antipodal map (multiplication by  $-1$ ).

(3) If  $n \neq 2$ , choose (for the duration of this paper) a minimal set of integers  $q_1, q_2, \dots, q_l$  prime to  $n$ , so that every primitive  $n^{\text{th}}$  root of unity is either some  $e^{(2\pi i/n)q_i}$  or a complex conjugate thereof. Then  $\tau$  splits into a Whitney sum  $\bigoplus_{i=1}^l \tau_i$  of complex bundles, so that each factor  $\tau_i$  is invariant under the action of  $\mathbf{Z}_n$ , and this restriction to each fiber of  $\tau_i$  is just complex multiplication by  $e^{(2\pi i/n)q_i}$ . Furthermore the splitting  $\tau \sim \bigoplus_{i=1}^l \tau_i$  is uniquely determined by the action  $r : \mathbf{Z}_n \times M \sim M$ . Let  $\{d_i : i = 1, 2, \dots, l\}$  be the complex dimensions of the fibers of  $\{\tau_i : i = 1, 2, \dots, l\}$ ; these numbers are equivalence (of  $\mathbf{Z}_n$ -actions) invariants. These results can be found on pages 104–107 of [3].

Let  $C_m(d_1, d_2, \dots, d_l)$  denote the set of equivalence classes of semi-free orientation preserving (differentiable)  $\mathbf{Z}_n$ -actions on  $S^m$ , where if  $n \neq 2$ , then  $d_i, i = 1, \dots, l$ , are as in (3) above; and if  $n = 2$ , then  $l = 1$ , and  $d_1$  equals the real dimension of the normal bundle  $\tau$  to  $F_r$ .  $C_m(d_1, \dots, d_l)$  has an abelian