

Asymptotic Estimates for Solutions of Linear Systems of Ordinary Differential Equations Having Multiple Characteristic Roots

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§1 Introduction. In a well known and by now classical paper [8], N. Levinson in 1948 gave a general theorem for asymptotically estimating a complete set of linearly independent solutions for a system of differential equations of the form

$$(1.1) \quad y'(t) = [A + V(t) + R(t)]y(t), \quad t \in (0, \infty),$$

where A is a constant square matrix with simple characteristic roots, $V(t) \rightarrow 0$ as $t \rightarrow \infty$ and is absolutely continuous on some interval $[t_0, \infty)$, and the elements of $R(t)$ are Lebesgue integrable. In 1954, M. A. Naimark, in his well known book on linear differential operators [9], showed how such types of asymptotic theorems could be used to obtain results on the difficult and important deficiency index problem for ordinary self-adjoint differential operators.

In 1966, one of us [3] (but see [4] for a corrected treatment), by generalizing and refining Naimark's techniques, was able to use Levinson's asymptotic theorem to compute the deficiency index of a wide class of self-adjoint differential operators. In the same year M. V. Fedorjuk [7] proved an asymptotic theorem for the solutions of a class of even order differential operators and used this to compute their deficiency indices. Fedorjuk's asymptotic theorem is not a consequence of Levinson's theorem, nor does it contain the latter. In [5] an asymptotic theorem is given which contains Levinson's theorem and will also essentially yield Fedorjuk's theorem.

All of the asymptotic theorems mentioned above assume that the matrix A in (1.1) has only simple eigenvalues. However, recent investigations on the deficiency index of a certain class of fourth order differential operators has indicated the necessity of being able to obtain asymptotic theorems when A has multiple eigenvalues. Such types of theorems were obtained in [6], [10], and [11] for special fourth order systems in which A has two simple eigenvalues and one double eigenvalue.