

The Characteristic Functions of Operator Theory and Electrical Network Realization

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One of the main results in Nagy and Foias [N-F] says that any analytic function $\theta(z)$ which is defined on the unit disk and whose values are operators of norm less than or equal to one on a Hilbert space coincides with the characteristic function of some operator T on another Hilbert space \mathbf{H} (Theorem VI, 3.1 [N-F]). Amazingly enough a very special case of this abstract theorem was developed independently by engineers and is one formulation of the main realizability theorem in electrical engineering. The purpose of this note is to sketch very roughly how this occurs.

The fact that a connection exists between modern operator theory and electrical engineering has been known for several years. In his book [L] Livsic worked out the connection between his approach to characteristic functions for operators and network realizability theory. Unfortunately, this book has never been translated into English and it has circulated little among American operator theorists. Also, DeWilde in [D] relates mathematics developed by Livsic and Potopov to network synthesis. There has been work by Sandburg, Saeks and Levan in a very different direction involving the original Halmos dilation for a contraction operator and the synthesis of a lossy network with a lossless network and a resistor.

This paper takes the Lax-Phillips [L-P] and Nagy-Foias viewpoint to operator theory rather than that of Livsic and the interpretation given here to the Nagy-Foias representation theorem appears to be new.

The first section is an exposition intended for operator theorists of some basic electric network theory, in the second part we relate this to operator theory. For a description of electric network theory see [W] or [N].

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A brief exposition of electrical network theory. An electric network (n port) is thought of as a box with n pairs of terminals on it. For simplicity sake consider a network \mathfrak{N} which is a 1 port. If one connects a voltage source with voltage v across the terminals of \mathfrak{N} , then a unique current i flows through the circuit (see Figure 1). Voltage and current are