

A Local Compactness Theorem for Wave Propagation Problems of Classical Physics

JOHN R. SCHULENBERGER

Communicated by RALPH PHILLIPS

1. Introduction. In many of the wave propagation phenomena of classical physics the functions describing the properties of the medium are not smooth but rather only piecewise continuous. When light passes through a lens, for example, the dielectric tensor in the system of Maxwell's equations describing this event has a jump discontinuity at the boundary of the lens. Because of this lack of smoothness, such basic physical phenomena do not fit into the scattering theory for symmetric hyperbolic systems developed earlier by C. H. Wilcox and the author [16, 17, 21]. The result needed here is a local compactness theorem of the type proved in §2 below.

For the purposes of scattering theory, the relevant local compactness theorems have usually been obtained via coerciveness inequalities [6, 7, 8, 10, 19]. However, the appropriate coerciveness results seem to require ellipticity or smooth coefficients or both (*cf.* [18]) and, in addition, special boundary conditions for problems in an exterior domain [7, 10, 12, 19]. Recently it has been possible in some cases to establish local compactness without coerciveness [1, 3, 11]. In particular, M. Š. Birman [1] has proved a local compactness theorem which enables him to include symmetric hyperbolic systems of constant deficit (*cf.* below) on the whole space in his abstract stationary theory, provided that the coefficients of the system are continuous. It is the purpose of this note to prove a local compactness theorem for systems of constant deficit on the whole space with coefficients which may be discontinuous.

Since this work was completed, V. G. Deĭč [3] has published results which include Theorem 3.1 below. This work is based on a rather general local compactness theorem, but this theorem requires a hypothesis in addition to those needed for Theorem 2.1 (*cf.* the remark below following this theorem), and his proof is quite different.

The paper is organized as follows. In §2 the local compactness theorem is formulated and proved. In §3 some applications to scattering theory are mentioned.