

# *Boundary Behavior of Non-parametric Minimal Surfaces*

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1. It is well known that the construction of barriers is an essential step in the proof of the existence and regularity of the solutions to the Dirichlet problem for nonlinear, non-uniformly elliptic equations.

In particular, Jenkins and Serrin ([3]) have considered the equation of non-parametric minimal surfaces in an open set  $\Omega$  in  $\mathbf{R}^n$ , and using the barrier technique have proved that there exists a unique solution to the minimal surface equation in  $\Omega$ , which takes prescribed values  $\varphi(x)$  on the boundary of  $\Omega$ , provided  $\partial\Omega$  is a  $C^3$ -hypersurface with non-negative mean curvature, and the function  $\varphi$  is twice-continuously differentiable. This solution is twice-continuously differentiable in all of  $\bar{\Omega}$ . Also, if  $\partial\Omega$  and the boundary data are more regular, say  $C^{k+\alpha}$  with  $k \geq 2$ , then the solution is  $C^{k+\alpha}$  in all of  $\bar{\Omega}$ . On the other hand, by means of an approximation technique (see [4]), one can show the existence and the uniqueness of a continuous non-parametric minimal surface, provided the function  $\varphi(x)$  is continuous and the condition on the mean curvature of  $\partial\Omega$  is satisfied.

The *a priori* gradient estimate of Bombieri, De Giorgi and M. Miranda ([1]) implies that this solution is analytic in  $\Omega$ , and hence is a solution to the Dirichlet problem for the equation of minimal surfaces with continuous boundary data.

The aim of this paper is to study the global regularity (*i.e.*, regularity up to the boundary) of minimal surfaces in the intermediate case in which the function  $\varphi(x)$  is more than merely continuous, but is not twice differentiable. We get the following results:

- a) If  $\varphi$  belongs to  $C^{1+\alpha}$ ,  $0 < \alpha \leq 1$ , then the solution  $u(x)$  is Lipschitz-continuous up to the boundary.
- b) If  $\varphi$  is Lipschitz-continuous, then  $u(x)$  is Hölder-continuous in  $\bar{\Omega}$ , with some positive exponent  $\sigma$ .

The statement (b) can be improved if we make an additional assumption on  $\partial\Omega$ . In fact, we have: