

Existence Theorems for Parametric Surfaces of Prescribed Mean Curvature

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1. Introduction. Let D be a domain in E^3 and let $H(x)$ be a given real valued function defined in D . In this work we treat the following problem: Given an oriented closed Jordan curve Γ contractible in \bar{D} , when can we find a surface of prescribed mean curvature $H(x)$ contained in \bar{D} which spans Γ in an orientation preserving fashion? That is, we seek a mapping $x: \bar{B} \rightarrow \bar{D}$, B denoting the two dimensional unit disc, satisfying the following conditions:

- (i) $x \in C^2(B) \cap C^0(\bar{B})$;
- (ii) x maps ∂B homeomorphically onto Γ with degree 1;
- (iii) x satisfies in B the equations

$$(1.1) \quad x_{uu} + x_{vv} = 2H(x)x_u \wedge x_v,$$

$$(1.2) \quad x_u^2 - x_v^2 = x_u \cdot x_v = 0.$$

Equations (1.2) state that x is a conformal mapping of B onto its image; under that condition equations (1.1) become the equations for mean curvature $H(x)$.

The case where D is convex and $H \equiv 0$ corresponds to the classical Plateau problem which was solved by Radó [21] and Douglas [5]. Heinz [9] considered the case of constant mean curvature H and showed that if D is a ball of radius $(17^{1/2} - 1)/(8|H|)$ then the problem is solvable. The radius was improved by Werner [29] to $1/(2|H|)$ and finally sharpened to the best possible value $1/|H|$ by Hildebrandt [13]. Hildebrandt has generalized this result to allow variable $H(x)$ [12] and also has considered the case where D is an ellipsoid [14]. In another direction, the authors [7] showed that if D is a cylinder of radius $1/(2|H|)$, then the problem is solvable; they also allowed variable $H(x)$.

These special results, although quite interesting in themselves, point to the need for a general theory to explain the phenomenon involved. This is the motivation for the problem we have considered. It is the object of this paper to provide such a general theory.

We shall show that it is the inward mean curvature of ∂D which is the crucial factor for the solvability of the problem. For example, in the case of constant