Ideal Sets and Ideals of Compact Operators on Hilbert Space

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1. Introduction. In this paper we pursue the study of the two sided ideal structure of the ring $\mathfrak{L}(\mathfrak{H})$ of all bounded linear operators on a separable infinite dimensional complex Hilbert space $\mathfrak{H}$, employing the same approach as in [1]. The fundamental tool of this approach (used implicitly throughout the present paper) is a theorem of Calkin [2, §1] which states the existence of a one to one lattice preserving correspondence between the two sided ideals of $\mathfrak{L}(\mathfrak{H})$ and certain subsets of the set of all bounded sequences of non-negative real numbers called ideal sets.

Definition. A set J of bounded sequences of non-negative numbers is called an ideal set if it satisfies the following conditions:

i) if $\{\lambda_n\} \in J$, then $\{\lambda_{\pi(n)}\} \in J$ for every permutation π of the set Z^+ of positive integers;
ii) if $\{\lambda_n\} \in J$ and $0 \leqq \mu_n \leqq \lambda_n$, for every $n \in Z^+$, then $\{\mu_n\} \in J$;
iii) if $\{\mu_n\} \in J$ and $\{\lambda_n\} \in J$, then $\{\mu_n + \lambda_n\} \in J$.

We recall that, if $\mathcal{J}$ is a proper two sided ideal of $\mathfrak{L}(\mathfrak{H})$, then the ideal set J, which corresponds to $\mathcal{J}$ under the above mentioned one to one mapping is the set of all sequences of eigenvalues (counted according to multiplicity) of all positive operators of $\mathcal{J}$.

The simplest non trivial examples of ideal sets are the set C of all sequences of non-negative numbers tending to zero (corresponding to the ideal of all compact operators on $\mathfrak{H}$) and the subset F of C consisting of all sequences such that only finitely many terms are different from zero (corresponding to the ideal of all finite rank operators on $\mathfrak{H}$). C and F turn out to be the maximal and the minimal non trivial ideal sets, respectively [2]. From now on, an ideal set J will be called proper if the inclusions $F \subset J \subset C$ are proper. The classical examples of proper ideal sets are the ideal sets C_p of the p-Schatten ideals

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