

Toeplitz Operators and Division by Inner Functions

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In this paper we study Toeplitz operators on certain weighted sequence spaces. We consider the Hilbert space of all sequences square summable with respect to a nondecreasing sequence of positive weights. It is shown that the complex conjugate of a bounded analytic function in the unit disk induces a bounded Toeplitz operator on this space. This implies the following result on an analytic function: If the sequence of Fourier coefficients of a function in the Hardy class H^2 is square summable with respect to a weight of the above type, then the sequence of Fourier coefficients of its outer part has the same property. From this theorem we can deduce a recent theorem of Korenbljum [6] on the division of analytic functions with derivatives in H^2 by inner functions. It is also possible to obtain a simple proof of a result on the division of analytic functions with finite Dirichlet integral by inner functions. This last result follows also from the representation formula for the Dirichlet integral due to Carleson [2].

Some operator theoretic problems concerning Toeplitz operators on weighted sequence spaces are studied in §2.

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§1. Let $H^p(1 \leq p \leq \infty)$ denote the Hardy spaces of analytic functions in the unit disk in the complex plane. $L^p(1 \leq p \leq \infty)$ will denote the Lebesgue spaces of the unit circle with Haar measure. The norm in L^p will be denoted by $\|\cdot\|_p$. The correspondence that sends a function in H^p to its boundary value function is an isometric isomorphism of H^p onto the subspace of L^p consisting of those functions whose negative Fourier coefficients vanish. (For these and other results on H^p theory, we refer the reader to Hoffman's book [5].) We can thus regard H^p as a subspace of L^p . We shall use the same notation for a function in H^p and its boundary value function.

Let P be the orthogonal projection of L^2 onto H^2 . If φ is in L^∞ the Toeplitz operator T_φ is defined as the (bounded) operator on H^2 that sends f in H^2 to $P(\varphi f)$. Basic properties of these operators can be found in Halmos [4, Chapter 20].