

*Existence of Solutions of the Hartree
Equations for N Electrons
An Application of the Schauder-Tychonoff
Theorem*

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I. Introduction. This paper will deal with the existence of solutions of the Hartree equations for N electron atoms or ions. The Hartree equations arise as an approximation to the Schrödinger equation in the following way: Each electron is assumed to move in a spherically averaged potential due to the other electrons as well as the potential due to the nucleus. Schrödinger's equation is then solved for an electron moving in such a potential, an easy task on account of the spherical symmetry. Thus, each electron is treated separately and the corresponding wave function is found with the desired quantum number. Each wave function is normalized so that the integral of the square over all space of the wave function is unity. Physically this just means that the probability of finding a particular electron somewhere in space is one. These wave functions are then used in finding the charge density arising from each electron. The total charge density due to the electrons present in the atom is then used to calculate the total potential by electrostatics. The self-consistent field requirement is then imposed; that is, the final potential must agree with the initial one which has been assumed in the Schrödinger's equation for the individual electron.

The Hartree assumptions then lead to one-electron wave functions which have the form

$$\mu_{nlm_l}(r, \theta, \varphi) = \Theta_{lm_l}(\theta)\Phi_{m_l}(\varphi) \frac{p_{nl}(r)}{r}$$

where

$$\int_0^\infty p_{nl}^2(r) dr = 1.$$