

A Method of Symmetrization for Plane Condensers

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§1. Introduction. Let $\Omega = (D, E, F)$ be a plane condenser, that is, let D be a plane domain, E a compact set, $E \subset D$, and F the complement of D . We assume throughout this paper that the interior of E contains 0 and that (with the terminology of [4, Chap. 4]) Ω is admissible.

If $|z| < \rho$ is contained in D , then, following Marcus [7], we define

$$(1.1) \quad L_\rho(\phi) = \int_I \frac{dr}{r} \quad (0 \leq \phi < 2\pi)$$

where $I = I_\rho(\phi)$ is the intersection of D with $\{z: |z| \geq \rho, \arg z = \phi\}$, and

$$(1.2) \quad R(\phi) = \rho \exp L_\rho(\phi).$$

$R(\phi)$ is of course independent of ρ .

Let $\Omega_i = (D_i, E_i, F_i)$ ($i = 1, 2$) be two condensers as above. For $-\infty \leq t \leq \infty$ we define the t -mean symmetrized domain of D_1 and D_2 as

$$(1.3) \quad D(t) = \{z: z = re^{i\phi}, 0 \leq r < R(\phi, t), 0 \leq \phi < 2\pi\}$$

where

$$(1.4) \quad R(\phi, t) = \begin{cases} \min [R_1(\phi), R_2(\phi)], & t = -\infty, \\ \max [R_1(\phi), R_2(\phi)], & t = \infty, \\ [R_1(\phi)R_2(\phi)]^{1/2}, & t = 0, \\ [1/2 (R_1^t(\phi) + R_2^t(\phi))]^{1/t}, & 0 < |t| < \infty, \end{cases}$$

and $R_i(\phi)$ is given by (1.2) with $D = D_i$ ($i = 1, 2$). Thus $R(\phi; t)$ is the t -mean of $R_1(\phi)$ and $R_2(\phi)$ (see [3, p. 16] for the basic properties of t -means). The t -mean symmetrized condenser of Ω_1 and Ω_2 is defined as

$$\Omega(t) = (D(t), E(t), F(t))$$