

A Symmetrizer for Certain Hyperbolic Mixed Problems with a Singular Coefficient

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I. Introduction. We shall treat here a class of hyperbolic initial-boundary value problems in a half space in which the lower order term is allowed to increase exponentially in the direction normal to the boundary. Kreiss [3] has shown that a general type of mixed initial-boundary value problems in a region with a smooth boundary is L_2 well-posed. He treated strictly hyperbolic systems with smooth, bounded coefficients. Further work on this problem was done by Balaban [1], Ralston [6], Rauch [7], and Sakamoto [8].

In [5], we obtained well-posedness conditions for certain hyperbolic systems in regions with corners. A new technique in our paper involved the construction of a symmetrizer which itself satisfied a homogeneous differential equation. The condition for well-posedness involving the boundary operator was just that the boundary operator be uniformly invertible on the space of solutions to the homogeneous equation restricted to the boundary. This is both the Kreiss condition and also the analogous Lopatinskii condition for elliptic equations [4].

In this paper we shall discuss the symmetrizer, use it to rederive some of Kreiss' results in a simpler way, while extending these results to include certain very singular coefficients.

II. Statement of the problem, main theorem. The differential operator we shall consider is of the form:

$$(2.1) \quad Lu = \sum_{i=1}^n A_i u_{x_i} + B(x_1, x_2, \dots, x_n, t)u - u_t,$$

where u is a complex valued k vector, each A_i is a Hermitian complex constant matrix, B is a square matrix measurable complex valued function of all its variables subject to the conditions

$$(2.2) \quad \operatorname{Ess\,sup}_{x,t} \operatorname{Re} B(x, t) \leq CI,$$