

The Azumaya Algebra of Some Polynomial Closed Rings

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Introduction. In [4], F. DeMeyer studied the polynomial closure Ω of a weakly uniform ring R . The polynomial closure Ω is a locally separable sub-algebra of the separable closure defined by G. Janusz in [5] when the ground ring has no idempotents except 0 and 1. This paper studies the finitely generated and projective modules and the Azumaya algebras over the polynomial closure Ω of some kind of weakly uniform rings. It is proved that the Azumaya algebras over Ω are split algebras in case the stalks of the Pierce sheaf defined on $\text{Spec } B(R)$ are noetherian and semi-local ($B(R)$ = the set of idempotents of R , $\text{Spec } B(R)$ = the set of maximal ideals with the hull-kernel topology); this extends Theorem 4 in [3]. Also, a finitely generated and projective Ω -module is a finite direct sum of free Ωe_i -modules where e_i are in $B(R)$ with $\sum_{i=1}^n e_i = 1$ and $e_i e_j = 0$ for $i \neq j$; this result extends the fact on page 377 in [1]. In the latter half, the cancellation laws of Azumaya algebras and projective modules over Ω are proved.

Throughout we assume that R is a commutative ring with identity 1 and that all modules are unitary left modules over a ring or an algebra. Let $B(R)$ be the set of idempotents of R and let $\text{Spec } B(R)$ be the set of maximal ideals in $B(R)$ with the hull-kernel topology. This topology has a basis $\{U_e\} = \{x/x \text{ in } \text{Spec } B(R) \text{ and } 1 - e \text{ in } x \text{ for } e \text{ in } B(R)\}$. Then it is totally disconnected, compact and Hausdorff. Also from [9], the Pierce sheaf of the stalks $R_x = R/xR$ for x in $\text{Spec } B(R)$ is defined on $\text{Spec } B(R)$, and R is the global cross section of this sheaf. We denote $R_x \otimes_R M$ by M_x for an R -module M and x in $\text{Spec } B(R)$. As given in [4], a strongly separable R -algebra is a finite projective and separable R -algebra. A ring R is a uniform ring if for each x in $\text{Spec } B(R)$ there is a collection of isomorphisms $\phi_y : R_y \rightarrow R_x$ (y in $\text{Spec } B(R)$) such that if F is a finite subset of R there is a neighborhood U of x with $\phi_y(a_y) = a_x$ for all a in F , y in U . A weakly uniform ring is a finite direct sum of uniform rings. A polynomial $f(X)$ is called separable if it is monic and $R[X]/(f(X))$ is a separable