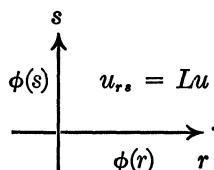


Goursat Problems for $u_{r,s} = Lu$

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Section 1. Introduction. Several types of Goursat problems have been considered in the literature: $u_{r,s} = f(r, s, u, u_r, u_s)$ [3, 8, 10]; $u_{r,s} = \sum u_{v_i v_i}$ [1, 6, 7, 10, 11, 13, 15, 16]; $u_{r_1, r_2, \dots, r_n} = f(r_1, r_2, \dots, u)$ [2], [14]; and recently $u_{r_1, r_2, \dots, r_n} = Lu$ where L is a linear operator (independent of r_1, \dots, r_n) [5]. The usual problem associated with these equations is that of finding solutions in the "interior" quadrant $r, s > 0$ or $r_1, \dots, r_n > 0$ which attain prescribed values on the quadrant's boundary; for example,



Conditions have been determined whereby these problems are wellposed (that is, solutions exist, are unique, and depend continuously on the boundary data). On the other hand solutions for the equation $u_{r,s} = \sum u_{v_i v_i}$ (which is related to the wave equation) do not in general exist in the "exterior" quadrant $r > 0, s < 0$ [13], [15]. (We note that this exterior quadrant problem corresponds to the interior quadrant problem for the equation $u_{r,s} = -\sum u_{v_i v_i}$.)

This paper considers interior, exterior and half-space ($s > 0, -\infty < r < \infty$) problems for the equation $u_{r,s} = Lu$, where L is a closed linear operator on a space X . In Theorem 2.1 we show the equivalence of the interior problem for $u_{r,s} = Lu$ with the second order Cauchy problem for $v_{i,i} = Lv$. This result is similar to some recent results of H. O. Fattorini [5] where the equivalence of wellposedness of interior Goursat and of Cauchy problems is presented. Section 2 concludes with a uniqueness result after imposition of a "growth" condition on the operator L . Corollary 3.1 shows that for the case where X is a Banach space the interior and exterior Goursat problems are (simultaneously) wellposed if and only if L is a bounded operator. Following three lemmas we obtain our basic result: Theorem 3.6. We find that when X is a locally convex sequentially