

Quasiconformal Mappings Near the Boundary

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1. Introduction and result. Let G be a finite, simply connected region with $0 \in G$ and $1 \in \partial G$, and assume that $z_0 \in G$ can be connected to $z = 1$ by a Jordan arc γ of length L lying in G except for its endpoint $z = 1$. Let f be a K -quasiconformal mapping of G onto the disc $D_w = \{w : |w| < 1\}$ with $f(0) = 0$ and $f(z) \rightarrow 1$ as $z \rightarrow 1$ on γ .

Theorem. *Under these assumptions we have*

$$(1.1) \quad |f(z_0) - 1| < 4^{2-1/K} \cdot L^{1/(2K)}.$$

If in addition $G \subset H$, where H is a half plane with $1 \in \partial H$, we have

$$(1.2) \quad |f(z_0) - 1| < 4^{2-1/K} \cdot L^{1/K}.$$

These results generalize a theorem of a previous paper [1] in two directions: If z_0 is visible from $z = 1$, *i.e.*, γ can be taken as the segment $(z_0, 1)$, and if f is a conformal mapping ($K = 1$), our theorem reduces to (1.1) and (1.2) of [1]. We also see that 4 cannot be replaced by a constant < 2 . The estimate (1.1) (for $K = 1$) answers a question raised by Helmut Wielandt.

2. An estimate of harmonic measure. Assume that the capacity of γ is $c < \frac{1}{4}$. We shall estimate the harmonic measure of γ with respect to $G \setminus \gamma$ at $z = 0$:

$$(2.1) \quad m(0, \gamma, G \setminus \gamma) \leq \frac{2}{\pi} \arcsin (2c^{1/2}).$$

Since the capacity c and the length L of γ satisfy $L \geq 4c$ (see Pommerenke [3], p. 66; I owe this reference to Dr. Hübner), this implies

$$(2.2) \quad m(0, \gamma, G \setminus \gamma) \leq \frac{2}{\pi} \arcsin (L)^{1/2}$$