

Remarks on the Pairings of Bredon, Milnor, and Milnor-Munkres-Novikov

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§1. Introduction. In this paper we study the pairings of Bredon, Milnor, and Milnor-Munkres-Novikov, denoted by $\rho_{n,k}$, $\sigma_{n,k}$, and $\tau_{n,k}$, respectively. We will give the definitions later in this section. Throughout we will be assuming that $n \geq k$ unless otherwise stated.

Γ^n denotes $\pi_0(\text{Diff } S^{n-1})$ with group operation induced by composition of diffeomorphisms. The isotopy classes may be replaced by pseudoisotopy classes and the diffeomorphisms may be assumed to fix pointwise a disk. For $n \geq 5$, Γ^n is equivalent to the group of homotopy n -spheres up to diffeomorphism under the operation of connected sum (denoted θ_n in [9]). We will use Γ^n in both ways; when it is used the second way we are implicitly assuming $n \geq 5$ or $\Gamma^n = \{S^n\}$ if $n < 5$.

Our principal result concerns $\rho_{n,k} : \Gamma^n \times \pi_{n+k}(S^n) \rightarrow \Gamma^{n+k}$. We relate $\rho_{n,k}$ to the diffeomorphism type of a product of homotopy spheres $\Sigma^n \times \Sigma^k$. Using this relationship and an invariant which we construct, we show that the bP_{n+k+1} component of the image of $\rho_{n,k}$ is zero in most cases when $n+k \equiv 3 \pmod{4}$. This essentially reduces computation of $\rho_{n,k}$ to a homotopy theory question in these dimensions since the π_{n+k}^S/J_{n+k} component depends only on compositions in stable homotopy (we are using the splitting $\Gamma^{n+k} \simeq bP_{n+k+1} \oplus \pi_{n+k}^S/J_{n+k}$ of [4]). As an application we show that the inertia subgroup of $\Sigma^n \times \Sigma^k$ has bP_{n+k+1} component equal to zero, $n+k \equiv 3 \pmod{4}$, generalizing a result of Schultz [16].

We also study the pairing $\tau_{n,k} : \Gamma^{n+1} \times \pi_k(\text{SO}(n)) \rightarrow \Gamma^{n+k+1}$ in the case $n+k+1 \equiv 1 \pmod{8}$. Here we show that the bP_{n+k+2} component (using the splitting of [5]) of the image of $\tau_{n,k}$ is zero. Finally, we relate the pairing $\sigma_{n,k} : \pi_n(\text{SO}(k+1)) \times \pi_k(\text{SO}(n)) \rightarrow \Gamma^{n+k+1}$ to $\tau_{n,k}$. Our result is that in a metastable range $\tau_{n,k}$ factors through $\sigma_{n,k}$.

We now give the definitions of the pairings. First we establish the usual orientation for $S^{n+k+1} \subset R^{n+1} \times R^{k+1}$. Then S^{n+k+1} has a decomposition $S^{n+k+1} = S^n \times D^{k+1} \cup (-1)^{n+1} D^{n+1} \times S^k$ where $(-1)^{n+1}$ indicates the orientation imposed