

Vector Fields on Quotient Manifolds of Spheres

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1. Introduction. Let M denote a smooth $(n - 1)$ -dimensional manifold and let $\sigma(M)$ denote the maximum number of independent vector fields on M . In this paper we will study the function $\sigma(M)$ for manifolds which are finitely covered by a homotopy sphere. In this case M has the form Σ^{n-1}/G where G is a finite group acting freely and differentiably on a homotopy sphere Σ^{n-1} . If Γ is a Sylow 2-subgroup of G it is a well known fact that Γ is either cyclic or a generalized quaternion group [10]. Let \mathbf{R} , \mathbf{C} , \mathbf{H} denote respectively the real, complex, or quaternionic numbers and define the *type* of G to be \mathbf{R} if Γ is \mathbf{Z}_2 or the trivial group, \mathbf{C} if Γ is cyclic of order at least 4, and \mathbf{H} if Γ is a generalized quaternion group. The Radon-Hurwitz number $\rho(n, F)$ is defined to be the largest integer k such that \mathbf{R}^n is a $\mathbf{C}_{k-1} \otimes F$ -module, where \mathbf{C}_{k-1} denotes the Clifford algebra on $k - 1$ generators. Our main result is the following.

Theorem 1.1. *Suppose $n \neq 8, 16$. If G has type F , then $\sigma(M) = \rho(n, F) - 1$.*

This extends the main result of [7] which dealt with the case where G acts linearly on the sphere S^{n-1} . In the case where the order of G is odd (1.1) has been obtained by Yoshida [27]. Sjerve [23] has given an example to show that the result is false when $n = 8$. It seems likely that it is true when $n = 16$, and we will show that this is the case except when G has type \mathbf{C} and the action of G on Σ^{15} is non-linear.

2. Invariance of $\sigma(M)$. If ξ is an n -plane bundle over a finite dimensional complex B let $\sigma(\xi)$ denote the maximum number of independent cross sections to ξ and let $\bar{\sigma}(\xi)$ denote the maximum of $\sigma(\xi')$ where ξ' is an n -plane bundle stably isomorphic to ξ . If M is a smooth n -manifold with tangent bundle $\tau(M)$ we abbreviate $\sigma(\tau(M))$ and $\bar{\sigma}(\tau(M))$ to $\sigma(M)$ and $\bar{\sigma}(M)$ respectively. We assume throughout that M is closed and connected.

The following result is due to W. Sutherland [24].