

Bessel Potentials. Inclusion Relations Among Classes of Exceptional Sets

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Communicated by W. P. ZIEMER

Introduction. Let $g_\alpha = g_\alpha(x)$ be the Bessel kernel of order α , $0 < \alpha < +\infty$, on \mathbf{R}^n ; g_α is the Fourier transform of $(2\pi)^{-n/2}(1 + |\xi|^2)^{-\alpha/2}$. If $1 < p < \infty$ we define a corresponding capacity $B_{\alpha,p}$, which we call a Bessel capacity. For $A \subset \mathbf{R}^n$,

$$B(A) = B_{\alpha,p}(A) = \inf_f \int f^p(x) dx,$$

the infimum being taken over all functions f in \mathcal{L}_p^+ (positive functions in the Lebesgue class) such that

$$g_\alpha * f(x) \geq 1 \quad \text{for all } x \in A.$$

These capacities have been studied in [9] and the present work leans heavily on that.

The null sets of the Bessel capacities are playing an increasingly important role in analytical investigations; for example they occur in the theory of functional completion [1] and in connection with removable singularities of solutions of differential equations [11], as well as in other contexts, as a means of describing small sets.

In the present work we are principally interested in the problem of finding inclusion relations between the classes of null sets of the Bessel capacities. More precisely, we say that Bessel capacity B is *stronger* than Bessel capacity B' , if $B(A) = 0$ always implies $B'(A) = 0$. If in addition, there is a set, A , such that $B(A) > 0$ but $B'(A) = 0$ we say that B is *strictly stronger* than B' . We call these, *relations* between the capacities B and B' and our goal is to find all the relations between the Bessel capacities.

Before describing our results let us begin with a description of results already in the literature. Various authors have considered the problem of finding necessary conditions and sufficient conditions that there exist a Riesz potential $|x|^{\alpha-n} * f$, where $f \in \mathcal{L}_p^+(1 < p < +\infty)$ and $\alpha p \leq n$, identically infinite on a