

Traces of Potentials. II.

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We continue the discussion begun in [1]. Specifically, we are interested in the non-linear potentials

$$g_\alpha * (g_\alpha * \mu)^{1/(p-1)}(x) \equiv U_{\alpha,p}^\mu(x), \quad 1 < p < \infty,$$

where μ is a (non-negative) Borel measure and g_α is the Bessel kernel of order $\alpha > 0$, i.e., $g_\alpha(x)$, $x \in \mathbf{R}^n$, is the Fourier transform of $(1 + |\xi|^2)^{-\alpha/2}$, $\xi \in \mathbf{R}^n$. More explicitly,

$$g_\alpha(x) = c_\alpha \int_0^\infty t^{(\alpha-n)/2} \exp[-(|x|^2/4t) - t] \frac{dt}{t}$$

where the constant c_α normalizes so that $\|g_\alpha\|_1 = 1$. The potentials $U_{\alpha,p}^\mu$ arise in the study of a non-linear analogue of the classical Newtonian capacity. See [2], [7] and [8]. The classical theory corresponds to the case $p = 2$, $\alpha = 1$.

As pointed out in [1], the regularity of $U_{\alpha,p}^\mu$ sheds light on the "trace" or restriction of potentials $g_\alpha * f(x)$ to subsets of \mathbf{R}^n of n -dimensional Lebesgue measure zero. Here f is an arbitrary function of the usual Lebesgue class L_p (* denotes convolution over \mathbf{R}^n). To measure the trace of $g_\alpha * f$ on a given compact exceptional set K , we look for measures $\mu \neq 0$, concentrated on K , such that the mapping $g_\alpha : L_p \rightarrow L_q(\mu)$ is continuous, i.e., there is a constant A independent of f in L_p such that

$$(1) \quad \left(\int |g_\alpha * f|^q d\mu \right)^{1/q} \leq A \|f\|_p, \quad 1 < p, q < \infty.$$

In analogy to the classical restriction theorems of functions in the Sobolev classes $W^{l,p}$ to hypersurfaces of dimension d contained in \mathbf{R}^n , we consider compact sets $K \subset \mathbf{R}^n$ with positive Hausdorff h -measure, $H_h(K) > 0$. $H_h(K) = \lim_{\epsilon \rightarrow 0} \inf \Sigma_\epsilon h(r_\epsilon)$, the infimum being over all countable coverings of K by balls of radius $r_\epsilon \leq \epsilon$, $\epsilon > 0$. Here $h = h(r)$ is a non-negative, non-decreasing, continuous function of $r \geq 0$, $h(0) = 0$. By a theorem of Frostman-Carleson [4], the existence of $\mu \neq 0$, concentrated on K , satisfying the growth condition