

Spectral Properties of Strictly Cyclic Operator Algebras

ALAN LAMBERT

Communicated by the Editors

1. Introduction. This paper is concerned with the spectra of operators on Hilbert space commuting with an algebra of finite strict multiplicity. It is shown that many properties of such operators can be deduced from consideration of the more easily analyzed strictly cyclic abelian algebras. We show that every operator commuting with an algebra of finite strict multiplicity has the same spectrum as an operator in an abelian strictly cyclic algebra (Corollary 3.2). It is then shown in Theorem 3.3 that every operator A in an abelian strictly cyclic algebra on an infinite dimensional Hilbert space admits a unique finite decomposition via mutually orthogonal reducing subspaces for A in such a way that all but one part has finite spectrum and the remaining part has spectral radius strictly less than its norm. As an application of this result we extend to the class of hyponormal operators a recent result of R. Bolstein and W. Wogen [1] concerning subnormal operators.

For general treatments of strict cyclicity and more generally finite strict multiplicity the reader is directed to [2], [3], [4], and [5].

2. Preliminaries. Let \mathcal{H} be a complex Hilbert space and let $\mathcal{L}(\mathcal{H})$ be the algebra of all bounded linear transformations from \mathcal{H} to \mathcal{H} (henceforth called operators). Throughout this paper the term "algebra" is reserved for subalgebras of $\mathcal{L}(\mathcal{H})$ which contain the identity operator I and are closed in the strong operator topology. When the notational situation dictates the identity operator on \mathcal{H} will be written as $I_{\mathcal{H}}$. As it causes no confusion whenever \mathcal{K} is a closed subspace of \mathcal{H} we will also write $I_{\mathcal{K}}$ for the orthogonal projection of \mathcal{H} onto \mathcal{K} .

A subalgebra \mathcal{A} of $\mathcal{L}(\mathcal{H})$ is defined to be an algebra of *finite strict multiplicity* (see [3]) if there is a finite set of vectors $\{x_1, x_2, \dots, x_n\}$ in \mathcal{H} such that $\mathcal{H} = \{A_1x_1 + \dots + A_nx_n : A_i \in \mathcal{A}, i = 1, \dots, n\}$. We will abuse notation somewhat and refer to the algebra of finite strict multiplicity $(\mathcal{A}; \{x_i\}_{i=1}^n)$. If $(\mathcal{A}; x)$ is an algebra of finite strict multiplicity we call it a *strictly cyclic* algebra. If moreover whenever A is in \mathcal{A} and $Ax = 0$ implies $A = 0$ we say that $(\mathcal{A}; x)$ is a *strictly*