

Estimates for Fully Developed Shock Solutions to the Equation

$$\frac{\partial u}{\partial t} - \frac{\partial v}{\partial x} = 0 \quad \text{and} \quad \frac{\partial v}{\partial t} - \frac{\partial \sigma(u)}{\partial x} = 0$$

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1. **Introduction.** Greenberg [1] recently announced a global existence theorem for the quasi-linear wave equation:

$$(1.1) \quad \frac{\partial u}{\partial t} - \frac{\partial v}{\partial x} = 0 \quad \text{and} \quad \frac{\partial v}{\partial t} - \frac{\partial \sigma(u)}{\partial x} = 0, \quad -\infty < x < \infty \quad \text{and} \quad t > 0.$$

Subsequently, it was learned that Bakvalov [2] obtained a similar result. The Greenberg Theorem was

Theorem 1. *Suppose σ is smooth and satisfies*

$$\sigma'(u) > 0, \quad \sigma''(u) < 0, \quad \lim_{u \rightarrow \infty} \int_1^u (\sigma'(s))^{1/2} ds = \infty$$

and

$$(1.2) \quad (\sigma'(w))^{1/2}(\sigma'(u))^{1/2} \geq \frac{\sigma(w) - \sigma(u)}{w - u}$$

for all $0 < u < w$. Then, if u_0 and v_0 are bounded measurable functions which are of locally bounded variation and satisfy

$$(1.3) \quad \begin{aligned} & \lim_{u \rightarrow 0^+} \int_1^u (\sigma'(s))^{1/2} ds < q \\ & = \frac{1}{2} \left[\inf_x \left(v_0(x) + \int_1^{u_0(x)} (\sigma'(s))^{1/2} ds \right) - \sup_x \left(v_0(x) - \int_1^{u_0(x)} (\sigma'(s))^{1/2} ds \right) \right] \end{aligned}$$

the initial value problem (1.1) together with the initial condition

$$(1.4) \quad \lim_{t \rightarrow 0^+} (u, v)(x, t) = (u_0, v_0)(x), \quad -\infty < x < \infty,$$