

The Asymptotic Behavior of the First Real Eigenvalue of a Second Order Elliptic Operator With a Small Parameter in the Highest Derivatives

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Communicated by MARK KAC

Introduction. Let L_ϵ be a second order elliptic operator with the highest coefficients multiplied by a parameter ϵ^2 (see (1.1)). Consider the first real eigenvalue λ_ϵ of the Dirichlet problem corresponding to $-L_\epsilon$ in a bounded domain Ω . Assume that there are compact subsets K_1, \dots, K_l of Ω such that the following holds: any characteristic curve of the first order operator L_0 that starts at a point of $\bar{\Omega}$ never leaves $\bar{\Omega}$ and its ω -limit set is one of the sets K_i . With each K_i one associates a number V_i which, intuitively, measures the amount of energy the limiting diffusion process ($\epsilon \rightarrow 0$) will require in order to cross from K_i to the boundary of Ω . The main purpose of this paper is to establish the following estimates:

$$(0.1) \quad \begin{aligned} \overline{\lim}_{\epsilon \rightarrow 0} (-2\epsilon^2 \log \lambda_\epsilon) &\leq \max (V_1, \dots, V_l), \\ \underline{\lim}_{\epsilon \rightarrow 0} (-2\epsilon^2 \log \lambda_\epsilon) &\geq \min (V_1, \dots, V_l). \end{aligned}$$

For $l = 1$ this result was recently announced by Ventcel [6]; a very brief outline of a proof is given in [6]. Our method of proof is different and appears to be much simpler. For $l > 1$, Ventcel [7] has announced a result which is inconsistent with (0.1); it is therefore fallacious.

We finally remark that the tools employed in this work (as well as in [6]) are probabilistic. Essential use will be made of probabilistic inequalities due to Ventcel and Freidlin [8].

1. The main results. Consider an elliptic operator