

Existence and Uniqueness of Positive Eigenfunctions for a Class of Quasilinear Elliptic Boundary Value Problems of Sublinear Type

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1. Following the classification which appears to have been introduced by Pimbley [14], [15], one calls the semi-linear boundary value problem

$$(1.1) \quad Lu = f(x, u), \quad x \in \Omega,$$

$$(1.2) \quad Mu = 0, \quad x \in \partial\Omega,$$

sublinear if the linear operator \mathfrak{L} determined by L and the boundary conditions (1.2) has, in some sense, a positive inverse and $0 < u f(x, tu) \leq t u f(x, u)$ for $x \in \Omega$, $u \neq 0$ and $t \geq 1$; the problem is called *superlinear* if \mathfrak{L} is the same and $0 < u f(x, tu) \leq t u f(x, u)$ for $x \in \Omega$, $u \neq 0$, and $0 < t < 1$.

If the operator \mathfrak{L} has a positive Green's function, so that (1.1)–(1.2) is equivalent to an integral equation

$$u(x) = \int_{\Omega} G(x, t) f(t, u(t)) dt,$$

then in the sublinear case one can expect uniqueness of the positive solution, *i.e.*, that (1.1)–(1.2) can have at most one solution which is positive in Ω . For the second order ordinary differential equation with zero end conditions such a result was proved by Picard [13]. (For this case the uniqueness proof reduces to showing that there is never a conjugate point in the interior of the interval. For the superlinear case, (see the discussion in [7]) the question of the uniqueness of a positive solution is a more subtle one.) In [17] Urysohn proved uniqueness of a positive solution for a sublinear case of the equation bearing his name, *i.e.*,

$$u(x) = \int_{\Omega} K(x, t; u(t)) dt.$$

Urysohn's result was abstracted and generalized by Krasnosel'skii and Lady-