

The Imbedding of Positive Hammerstein Integral Operators on Banach Lattices

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Communicated by G. BIRKHOFF

1. Introduction. In two recent papers [1-2], a novel gradient type technique has been proposed for the solution of the linear Fredholm integral equation

$$\phi(u) = g(u) + \int_x k(u, v)\phi(v) dv,$$

under a positivity assumption on the kernel. It was shown that the solution ϕ may be obtained in terms of the limiting behavior (in x) of the solution to the associated integral equation

$$\psi(u, x) = g(u) + \int_x \int_0^x \exp [-(x - y)/k(u, v)]\psi(v, y) dy dv.$$

Furthermore, it was pointed out how the function $\psi(u, x)$ can be expressed as a linear functional of the solution to a certain initial value problem. In passing it was also mentioned that the procedures of [1-2] formally carried over to Hammerstein equations, although appropriate conditions on the nonlinearity were not given.

The objectives of this paper are to extend in a precise way the above results to Hammerstein equations and to generalize the setting of the problem to order complete Banach lattices. The primary motivation for this type of setting is to point out explicitly that the fundamental results are basically order, rather than norm dependent.

2. Vector lattice extensions. In this section we shall give the vector lattice extensions of the results of [1] along with extensions to nonlinear problems. As indicated above, the motivation for this generalization is to simultaneously provide the most general useful setting for the method proposed and to explicitly indicate the order dependent nature of our results. Along the way, we shall give lattice versions of Gronwall's inequality and the monotone convergence theorem which may be of some interest in their own right.