

Two Limit Theorems for Random Differential Equations

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Communicated by MARK KAC

1. Introduction. Let (Ω, \mathcal{G}, P) be a probability space and, for $\epsilon > 0, t > 0$, let $V(\epsilon, t, \omega)$ be a linear operator (in general, unbounded) on a Banach space F . We assume that

$$(1) \quad \frac{dy^\epsilon(t)}{dt} = \epsilon V(\epsilon, t, \omega)y, \quad \text{for } t > s, \quad y^\epsilon(s) = f,$$

and under suitable regularity conditions, including a mixing condition, we obtain asymptotic formulas for y as $\epsilon \rightarrow 0$, and $t \rightarrow \infty$ in an appropriate scaling. Applications include wave propagation in random media, nonlinear stochastic ordinary differential equations, and the diffusion approximation in linear transport theory.

In equation (1) let

$$(2) \quad V(\epsilon, t, \omega) = V_1(t, \omega) + \epsilon V_2(t, \omega) + \epsilon^2 V_3(\epsilon, t, \omega)$$

where $\|V_j f\| \leq k(f)$ for $j = 1, 2, 3$, and f in a subspace \mathfrak{D} which is dense in F and is in the domain of $V(\epsilon_1, t_1, \omega_1) \cdot V(\epsilon_2, t_2, \omega_2) \cdot V(\epsilon_3, t_3, \omega_3)$ for all $\epsilon > 0, t > 0, \omega \in \Omega$. As usual, we suppress the argument ω when convenient. Here and below k always denotes a constant independent of ϵ and t .

For $f \in \mathfrak{D}$, let

$$(3) \quad \bar{V}_1(t_0, l)f = \frac{1}{l} \int_{t_0}^{t_0+l} E(V_1(t)) dt f,$$

$$(4) \quad \bar{V}_2(t_0, l)f = \frac{1}{l} \int_{t_0}^{t_0+l} E(V_2(t)) dt f,$$

$$(5) \quad \bar{V}_{11}(t_0, l)f = \frac{1}{l} \int_{t_0}^{t_0+l} \int_r^{t_0+l} E(V_1(s) V_1(r)) dr ds f.$$