

# Potential Kernels of Hunt Processes

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**Introduction.** It is well known that every Feller semigroup is also a Hunt semigroup (cf. [2] or [8]). Consequently, the classical theorem of Hunt on the existence of Feller semigroups (cf. [5]) gives a condition sufficient for a kernel  $V$  to be the potential kernel of a Hunt semigroup.

Let  $\mu$  be the measure on  $\mathbf{Z}$  such that, for all  $A \subset \mathbf{Z}$ ,  $\mu(A)$  is the cardinality of  $A \cap (\{0\} \cup \mathbf{N})$ . Then, if  $V$  denotes the corresponding convolution kernel, it is well known that it is the potential kernel of a Feller semigroup, namely the semigroup defining the Poisson process with mean holding time 1. However, it is easy to see that  $V$  does not satisfy the hypotheses of Hunt's theorem nor those of the generalizations of this theorem that are presented in [5] and [12].

The purpose of this article is to show that a certain class of kernels  $V$  (which includes the kernel defining the Poisson process) are the potential kernels of Hunt semigroups (Theorem 3.3). While this class by no means includes all Hunt kernels (for example, the identity kernel on  $\mathbf{R}^n$ ) it does include all kernels which arise "naturally" when constructing Hunt processes that correspond to axiomatic potential theories.

In the case of a suitable axiomatic potential theory on a locally compact space  $E$ , if the cone  $\mathcal{S}$  of excessive functions coincides with the cone  $\mathcal{C}$  of non-negative hyperharmonic functions, then  $V$  is the potential kernel of a Hunt semigroup providing (a) it satisfies the complete maximum principle and (b) for each compact  $K \subset E$ ,  $V1_K$  is a finite potential (in the sense of axiomatic potential theory) (Corollary 3.4). In particular, as is very well known, this is so if  $V(x, dy) = \|x - y\|^{2-n} dy$ ,  $E = \mathbf{R}^n$  ( $n \geq 3$ ) and the harmonic functions are taken to be the classical ones, in which case it is (up to a constant) the potential kernel of the semigroup defining Brownian motion.

In [4] Constantinescu showed, in the context of axiomatic potential theory, that if  $\mathcal{S} \subset \mathcal{C} \subset \mathcal{S}'$  (the cone of supermedian functions) and if  $V_\varphi$  is a potential for each  $\varphi \in \mathcal{C}_c^+(E)$  (this means that  $V$  is a nucleus in the terminology of [3]), then  $V$  is the potential kernel of a semigroup which is the transition semigroup of a strong Markov process. This process looks like a Ray process and, by the