

# *Stability of Parabolic Systems*

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**1. Introduction.** This paper is, in some sense, a development of an earlier paper [1] concerned with the application of Liapunov's direct method to the study of the stability of semilinear and quasilinear hyperbolic systems. In the present work these same ideas are applied to nonlinear parabolic systems of a fairly general type. To be precise, by establishing a series of fundamental lemmas and by appeal to Zubov's work, stability theorems are developed for second order quasilinear parabolic systems with respect to a suitable metric.

Starting with a linear constant coefficient system, known results are first rephrased and then extended to nonlinear systems in which the nonlinearity appears in a specially simple form. Then, by extending the system by augmenting it by some of the consequences obtained by its formal differentiation, the result is extended to cover a second order quasilinear parabolic system. It is, of course, necessary to require of the system that the functions concerned belong to the appropriate differentiability classes. The paper concludes with a simple application to Burgers equation which is an equation of some importance in continuum mechanics.

**2. Notation and boundary conditions.** We shall consider a quasilinear second order system of partial differential equations of the form

$$(2.1) \quad u_t = \Phi(u, x, t)u_{xx} + \Gamma(u, u_x, x, t),$$

where  $u(x, t) = \{u^i\}$  is an  $N \times 1$  vector with elements  $u^i = u^i(x, t)$  belonging to the respective classes  $C^1$  and  $C^2$  relative to  $x$  and  $t$ , the coefficient matrix  $\Phi(u, x, t) = \{\Phi^{ij}\}$  is an  $N \times N$  matrix and  $\Gamma(u, u_x, x, t) = \{\Gamma^i\}$  is an  $N \times 1$  vector. It will be assumed that the elements  $\Phi^{ij}$ ,  $\Gamma^i$  are defined and belong to the respective classes  $C^1$  and  $C^2$  relative to their arguments in the region

$$(2.2) \quad G \equiv \mathcal{J} \times [0, T],$$

with  $\mathcal{J}$  some closed interval in  $R^1$ . There will be no loss of generality in assuming the matrix  $\Phi$  to be symmetric.