

On the Kinematic Formula of Square of Mean Curvature Vector

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Communicated by S. S. CHERN

1. Introduction. Consider two closed surfaces of E^3 , one fixed and the other moving under the rigid motion g . Let the fixed one be M_0^2 and the moving one be gM^2 , and let dg be the kinematic density, so normalized that the measure of all positions about a point is $8\pi^2$. By taking the geometric invariants of the intersection of M_0^2 and gM^2 and integrating with respect to dg , we get so called kinematic formulas [1, 2]. The purpose of this paper is to use total square curvature as the integrand of the kinematic formula. As a special case of our general formula (9), we will prove the following formula:

Let $\tau(C)$ be the total square curvature of a curve C and for a surface M^2 in E^3 with mean curvature H and Gaussian curvature K , let

$$\bar{H}(M^2) = \int_{M^2} H^2 dv, \quad \bar{K}(M^2) = \int_{M^2} K dv, \quad \mu_0(M^2) = \int_{M^2} dv.$$

Our kinematic formula says

$$(1) \quad \int \tau(M_0^2 \cap gM^2) dg \\ = 2\pi^3(3\bar{H}(M_0^2) - \bar{K}(M_0^2))\mu_0(M^2) + 2\pi^3(3\bar{H}(M^2) - \bar{K}(M^2))\mu_0(M_0^2).$$

Our formula is not contained in formula (12) of [2], because the only invariant of $M_0^2 \cap gM^2$ introduced in [2] is the total volume $\mu_0(M_0^2 \cap gM^2)$ of $M_0^2 \cap gM^2$, *i.e.*, the length of the curve $M_0^2 \cap gM^2$. Our purpose is to show some kinematic formula which involves extrinsic invariants instead of intrinsic invariants χ, μ_* of [1, 2]. It is clear that not every geometric quantity can be integrated. For example, the integration of the total curvature $\int_{M_0^2 \cap gM^2} |k(s)| ds$ with respect to dg is very complicated and the integration of $\int_{M_0^2 \cap gM^2} k(s)^4 ds$ with respect to dg is even divergent.

2. Preliminaries. Let M^p, M^q be a pair of closed submanifolds in euclidean space E^n , where $p + q \geq n + 1$ so that generically $M^p \cap gM^q$ is a submanifold