

Local Behavior of Subharmonic Functions

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Communicated by MAURICE HEINS

1. Let u be a function subharmonic in the neighborhood of the origin in the complex plane \mathbf{C} , and let M be the concentrated mass at the origin of the associated negative mass distribution. A well known elementary result, correlating M with the growth rate of u at 0, is

$$(1.1) \quad \lim_{r \rightarrow 0} \frac{\mu_r u(0)}{\log(1/r)} = M,$$

where $\mu_r u(0)$ denotes the circumferential mean of u over the circle of radius r about the origin. We propose to generalize this result by showing that $\mu_r u(0)$ can be replaced by the function $u(re^{i\theta})$ itself, *i.e.*, that

$$(1.2) \quad \lim_{r \rightarrow 0} \frac{u(re^{i\theta})}{\log(1/r)} = M$$

uniformly, provided either θ or r omits a suitable exceptional set. In particular, (1.2) holds for almost all θ .

The following two theorems are established.

Theorem 1.1. *For any function u subharmonic in the neighborhood of the origin there exists a Borel G_δ set A on the unit circle such that A has logarithmic capacity zero and (1.2) holds uniformly over the set of all θ such that $e^{i\theta} \notin A$.*

Theorem 1.2. *For any function u subharmonic in the neighborhood of the origin there exists an open set B on the unit interval such that*

$$(1.3) \quad \int_B \frac{1}{r^2} dr < +\infty$$

and

$$(1.4) \quad \lim_{\substack{r \rightarrow 0 \\ r \notin B}} \frac{u(re^{i\theta})}{\log(1/r)} = M$$

uniformly in θ .