

Asymptotic Behavior of Solutions of Certain Elliptic Equations Near a Singular Manifold

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I. Introduction. We are concerned in this paper with the behavior as $t \rightarrow +\infty$ of solutions of elliptic equations of the form

$$(1) \quad \frac{\partial^2 u}{\partial t^2} + \frac{\mu}{t} \frac{\partial u}{\partial t} + t^{-2\alpha} \Delta u = k^2 u,$$

where Δ is the Laplace operator on a Riemannian manifold \mathfrak{M} of dimension N . If R_+ is the half-line $t > 0$ and $\mathfrak{M}_1 = R_+ \times \mathfrak{M}$, with the metric

$$(ds_1)^2 = t^{2\alpha} (ds)^2 + (dt)^2,$$

where ds is the metric on \mathfrak{M} , the above equation takes the form

$$(2) \quad \Delta_1 u + \frac{c}{t} \frac{\partial u}{\partial t} = k^2 u,$$

where Δ_1 is the Laplacian on \mathfrak{M}_1 and $\mu = N\alpha + c$.

The manifold $\mathfrak{M}_\infty = \{+\infty\} \times \mathfrak{M}$ is a singular manifold for equation (2) if $k^2 \neq 0$. If the metric of \mathfrak{M} is uniformly positive definite and if $\alpha \neq 0$, then (2) is not uniformly elliptic near \mathfrak{M}_∞ .

Thus our problem contrasts with those treated by Serrin and Myers (see [19] and [20]), where it is assumed that the equation is uniformly elliptic in the neighborhood of a singular point and that the coefficients of u and its first derivatives approach zero sufficiently rapidly near the singular point.

The case $\alpha = 1$, $c = 0$, $k^2 = -m^2 < 0$, $\mathfrak{M} = S_N$ (the N -sphere), was studied by Müller (see [15], [16]), who obtained results important for scattering theory. If $N = 2$, then

$$\lim_{r \rightarrow \infty} r^2 \int_r^{r+1} \int_{\mathfrak{M}} |u(t, x)|^2 dx dt$$

exists, and if it is finite, then as $t \rightarrow \infty$ we have essentially

$$u(t, x) \sim t^{-1} \{ e^{imt} f(x) + e^{-imt} f(-x) \}$$