

# *Local Nondeterminism and Local Times of Gaussian Processes*

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**1. Introduction.** This work grew from a study of the general conditions under which a Gaussian process has a smooth local time function. It is shown here that the main calculation in our previous work on the local time (*e.g.*, [1], [3], [4]) involves a property of Gaussian processes which is of independent interest—local nondeterminism. A process is *locally nondeterministic* if a future observation is “relatively unpredictable” on the basis of a finite set of observations from the immediate past. In other words, there is an unremovable element of “noise” in the local evolution of the sample function. We expect a random function with this property to be locally irregular. And so it is: We show that local nondeterminism is one of the two main sufficient conditions in our general result on the existence and smoothness of the local time of a general Gaussian process (Section 8). This in turn implies the local irregularity of the sample functions of the process.

Much of this paper is a systematic investigation of the conditions under which a Gaussian process is locally nondeterministic. Necessary and sufficient conditions are furnished in Section 2. These conditions are applied to various types of processes in Section 3. A major part of the work is devoted to the process with stationary increments (Sections 4–7): usable sufficient conditions are included in Sections 6 and 7.

The results of this study extend many of the sample function properties established in [1], [2], [3], [4] to more general kinds of processes.

**2. Local nondeterminism: definition and characterizations.** Throughout this paper  $X(t)$ ,  $-\infty < t < \infty$ , is a Gaussian process with mean 0. As is well known, the best (linear) predictor of the future value  $X(t+h)$ , for  $h > 0$ , based on the past data  $X(s)$ ,  $-\infty < s \leq t$ , is the conditional expectation  $E\{X(t+h) \mid X(s), -\infty < s \leq t\}$ . The *prediction error of lag  $h$*  is the conditional variance,  $\text{Var}\{X(t+h) \mid X(s), -\infty < s \leq t\}$ . The process is nondeterministic if the latter is positive for  $h > 0$  (*cf.* [5], p. 107).