

Initial-value Problems for Pseudo-parabolic Partial Differential Equations

V. R. GOPALA RAO & T. W. TING

Communicated by HANS LEWY

1. Introduction. We shall be concerned with an equation of the form,
(1.1)
$$(A - I)u_t + Bu = f,$$

for an unknown function $u(x, t)$ defined in the whole space $R^m \times R$, where A and B are second-order differential operators with coefficients being functions of $x \in R^m$ and $t \in R$. In what follows A will be assumed to be uniformly elliptic in $R^m \times R$ and the coefficient of u_t to be strictly negative. For earlier works on equations of this type, see the references in [29] where mixed problems in a cylindrical domain have been studied. Subsequently, various generalizations of the results in [29] have been obtained, [18], [28] and certain mildly non-linear equations of this kind have also been considered, [7], [12], [28].

Although many physical problems are governed by equations of the form in (1.1), the most significant application is to the two temperature theory for heat conduction, [6]. Based on this theory, various questions can be raised about the solution of (1.1). Here, we only generalize the results in [26] and [27] by considering the initial-value problem of finding a solution u of (1.1) in $R^m \times R$, which assumes prescribed values at a given instant, say $t = 0$.

It turns out that physical theory [6] requires that the coefficient of u_t in (1.1) be strictly negative. This together with the ellipticity of A ensures the existence of the principal fundamental solution of the equation $(A - I)u = 0$. In §2, basic properties of the principal fundamental solution have been studied. Some of these results are new and some old. In either case, these results have been stated in the form suitable for the present applications. Results in §2 are then applied in §3 to obtain an $W^{2,p}$ -estimate for the solutions of the equation $(A - I)u = v$ in the whole space. In §4, the initial value problem is reduced to solving an integro-differential equation which is then solved by successive approximations via the estimate obtained in §3. Finally, the regularity of the solution is studied in §5 under further restrictions on the initial data and on the coefficients in (1.1).