

# On The Stepanov-Almost Periodic Solution of an Abstract Differential Equation

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1. Let  $J$  be the interval  $-\infty < t < \infty$ ,  $X$  a Banach space and  $X^*$  the dual space of  $X$ . A continuous function  $f : J \rightarrow X$  is said to be (strongly) almost periodic if, given  $\epsilon > 0$ , there is a positive real number  $\ell = \ell(\epsilon)$  such that any interval of the real line of length  $\ell$  contains at least one point  $\tau$  for which

$$(1.1) \quad \sup_{t \in J} \|f(t + \tau) - f(t)\| \leq \epsilon.$$

We say that a function  $f : J \rightarrow X$  is weakly almost periodic if the scalar-valued function  $\langle x^*, f(t) \rangle = x^*f(t)$  is almost periodic for each  $x^* \in X^*$ .

For  $1 \leq p < \infty$ , a function  $f \in L^p_{loc}(J; X)$  is said to be Stepanov-bounded or  $S^p$ -bounded on  $J$  if

$$(1.2) \quad \|f\|_{S^p} = \sup_{t \in J} \left[ \int_t^{t+1} \|f(s)\|^p ds \right]^{1/p} < \infty.$$

For  $1 \leq p < \infty$ , a function  $f \in L^p_{loc}(J; X)$  is said to be Stepanov-almost periodic or  $S^p$ -almost periodic if, given  $\epsilon > 0$ , there exists a positive real number  $\ell = \ell(\epsilon)$  such that any interval of the real line of length  $\ell$  contains at least one point  $\tau$  for which

$$(1.3) \quad \sup_{t \in J} \left[ \int_t^{t+1} \|f(s + \tau) - f(s)\|^p ds \right]^{1/p} \leq \epsilon.$$

Let  $\mathfrak{L}(X, X)$  be the Banach space of all bounded linear operators on  $X$  into itself, with the uniform operator topology. An operator-valued function  $T : J \rightarrow \mathfrak{L}(X, X)$  is called a (strongly) continuous group if

$$(1.4) \quad T(t_1 + t_2) = T(t_1)T(t_2) \quad \text{for all } t_1, t_2 \in J;$$

$$(1.5) \quad T(0) = I = \text{the identity operator on } X;$$

$$(1.6) \quad \text{for each } x \in X, T(t)x, t \in J \rightarrow X \text{ is continuous.}$$