

Erratum: An Application of the Holomorphic Functional Calculus to Sharpen a Classical Matrix Commutator Theorem

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Communicated by the Editors

A counterexample in three dimensions to Theorem III of [2] as stated has been shown to the author by E. T. Kehlet and by B. K. Richard (see [1], the remark following the proof of Theorem 2).

The following correct statement of Theorem III (which is suggested in §2.1 of [1]) does not include Theorem I of [2] (this result of Marcus and Thompson is successfully generalized in [1]). It is perhaps of some interest, though, if only because the method of proof—linearization of group commutators to algebra commutators, combined with a result of C. R. Putnam for algebra commutators (Theorem II of [3], stated as Theorem II in [2])—seems to be the only method available.

Theorem III. *Suppose that A and B are operators on a Hilbert space. Assume that*

- (i)' $0 \notin \text{conv}(\text{spectrum } B)$;
- (ii)'' $B^n A B^{-n}$ commutes with A for all $n = 1, 2, \dots$;
- (iii)' A is normal.

Then B commutes with A .

Proof. The only modification to be made to the argument given in [2] is that hypothesis (ii)'' introduced above must be used to show that $(\log \Phi_B)(A)$ commutes with A , where notation is as in [2]. (It is at this point, in lines 32 and 33 of page 408 of [2], that a gap exists.)

As stated in [2], $\log \Phi_B$ is the limit (in norm) of rational functions of Φ_B , i.e., of maps $(\lambda - \Phi_B)^{-1}$ where λ is in the resolvent set of Φ_B . For $|\lambda| > \|\Phi_B\|$ the power series expansion

$$(\lambda - \Phi_B)^{-1} = \lambda^{-1}(1 + \lambda^{-1}\Phi_B + \lambda^{-2}\Phi_B^2 + \dots)$$