

Classification of Hamiltonian Linear Systems

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1. Introduction. A *hamiltonian linear system* is a system of first order differential equations in $2n$ variables of the form:

$$(1.1) \quad \begin{cases} \dot{p}^k = -\frac{\partial H}{\partial q_k} \\ \dot{q}_k = \frac{\partial H}{\partial p^k} \end{cases} \quad k = 1, \dots, n$$

where H is a *quadratic form* in the p 's and the q 's, and therefore $\partial H/\partial q_k$, $\partial H/\partial p^k$ are *linear functions*. H is called the *hamiltonian function* of the system. Two such systems are *equivalent* iff there is a linear transformation of the p 's and the q 's which carries one into the other; we shall call such an equivalence *symplectic similarity*.

In 1936, J. Williamson [1] obtained a complete classification of hamiltonian linear systems up to symplectic similarity. Both the results and the method are difficult to penetrate and they have remained relatively unknown to applied mathematicians and physicists. On the other hand, the original problem has been stated more recently in the wider context of the theory of bilinear forms and in that of algebraic groups, leading to several interesting works by pure algebraists [2, 3, 4]. These works are very elegant, but still impenetrable to non-specialists.

In this paper we present a reformulation of Williamson's classification problem which should be accessible to physicists and applied mathematicians, since in the development little more is required than the classical results of linear algebra. Our method should be considered as a compromise between Williamson's matrix approach and the sophisticated algebraic one that can be extracted from recent works.

2. Formulation of the problem and results. A *symplectic space* is a pair (W, ω) consisting of a linear space W over a field K , and a *symplectic form* ω (*symplectic* = anti-symmetric, non-degenerated, bilinear). A *symplectic subspace*