

# *A Property for Certain Derivatives*

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**Introduction.** The problem of determining when a real valued function of a real variable has a primitive is still unsolved. The first step towards resolving a question of this nature is to find properties possessed by every derivative. The best known of these is the intermediate value property or Darboux property. Less well-known but easy to prove is that every derivative is a function of Baire class one. In 1916, Denjoy enlarged the list of properties of derivatives, and in 1950, Zahorski proved a property for derivatives which includes Denjoy's.

In this work there is introduced a property of derivatives stronger than Zahorski's. It is shown that this property is also a long way from classifying derivatives by showing that this property is also possessed by every approximate derivative and by every  $k^{\text{th}}$  Peano derivative. It follows by a technique given in [1], that this property is also possessed by every  $k^{\text{th}}$   $L_p$  derivative for  $p \geq 1$ .

**Definitions and conventions.** All functions will be real valued functions of a real variable and measurable.

**Definition.** A function  $f$  has an approximate derivative  $f_{ap}'(x)$  at a point  $x$  if there is a measurable set  $E$  whose density at  $x$  is one such that

$$E - \lim_{y \rightarrow x} (f(y) - f(x))/(y - x) = f_{ap}'(x)$$

where  $E - \lim_{y \rightarrow x}$  means  $y$  must be in  $E$ .

For a concise development of the properties of approximate derivatives see [2].

**Definition.** Let  $k$  be a positive integer. A function  $f$  has a  $k^{\text{th}}$  Peano derivative at  $x$  if there are numbers  $f_1(x), \dots, f_k(x)$  such that

$$f(x + h) = f(x) + hf_1(x) + \dots + (h^k/k!)f_k(x) + o(h^k).$$

The number  $f_k(x)$  is called the  $k^{\text{th}}$  Peano derivative of  $f$  at  $x$ . The elementary properties of  $k^{\text{th}}$  Peano derivatives can be found in [3].