

On Para-Unicellular Operator Algebras

AVRAHAM FEINTUCH

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1. Introduction. Let H be a complex Hilbert space. A linear manifold $L \subseteq H$ is an *operator range* if there exists a Hilbert space K and a bounded linear transformation $T : K \rightarrow H$ such that $L = TK$. The idea of studying the invariant operator ranges of an algebra of operators was introduced by Foias in [2]. If \mathfrak{A} is a weakly closed algebra containing the identity, then \mathfrak{A} is para-unicellular if the invariant operator ranges of \mathfrak{A} form a chain. Foias showed that if \mathfrak{A} is para-unicellular, then every invariant operator range of \mathfrak{A} is closed. In this paper, it is shown that, in fact, every finite inflation of a para-unicellular algebra has this property.

If \mathfrak{A} is a weakly closed algebra containing the identity on H , then the closed invariant subspaces of \mathfrak{A} will be denoted by $\text{Lat } \mathfrak{A}$ and the invariant operator ranges of \mathfrak{A} by $\text{Lat}_{1/2} \mathfrak{A}$. If F is a family of closed subspaces of H , then $\text{Alg } F$ will be the algebra of bounded linear operators which leave every member of F invariant. Then \mathfrak{A} is *reflexive* if $\mathfrak{A} = \text{Alg Lat } \mathfrak{A}$. All para-unicellular reflexive algebras were classified by Foias in [2]. Here it is shown that if \mathfrak{A} is para-unicellular, under certain conditions $\mathfrak{A}^{(2)}$ is reflexive. Foias' theorem is then obtained as a corollary.

2. Preliminaries. If H is a Hilbert space and n is a positive integer, then $H^{(n)}$ denotes the orthogonal direct sum of n copies of H . If A is any operator on H , $A^{(n)}$ denotes the direct sum of n copies of A acting on $H^{(n)}$ in the standard fashion. If \mathfrak{A} is an algebra of operators on H , then $\mathfrak{A}^{(n)} = \{A^{(n)} : A \in \mathfrak{A}\}$. \mathfrak{A}' is used to denote the commutant of \mathfrak{A} .

The next lemma is well known and has been a major tool used in the study of the relationship between algebras of operators and their invariant subspaces.

Lemma 2.1. *Let \mathfrak{A} be an algebra of operators on H which contains the identity. Then the weak closure of $\mathfrak{A} = \{B : \text{Lat } \mathfrak{A}^{(n)} \subseteq \text{Lat } B^{(n)} \text{ for all integers } n \geq 1\}$.*

Proof. See [3].

3. An example. Most of the results of this paper were motivated by the following matrix algebra on \mathbf{C}^3 .