

A New Example of a Jacobson Radical Simple Ring

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For a number of years, one of the outstanding problems in radical theory was the existence of a Jacobson radical simple ring. E. Sasiada announced the existence of such a ring around 1961, and he and P. M. Cohn published the example in 1967 [3]. While Sasiada invented a ring with the desired properties, this paper presents a new example which is a homomorphic image of an ideal of the algebra $B(H)$ of operators on a separable infinite dimensional Hilbert space H .

All the necessary background material is contained in "Semiprime Ideals and Irreducible Ideals of the Ring of Bounded Operators on Hilbert Space" by D. Morris and N. Salinas [2]. The first theorem below comes directly from that paper.

Theorem. *If I is an ideal of $B(H)$, then I is semiprime iff $I = I^p$ where p is any positive number not equal to one.*

Theorem. *Let A be a compact, positive, diagonal operator which is not of finite rank. Suppose that the ideal $\langle A \rangle$ generated by A is semiprime. Then there exists an ideal M maximal with respect to missing $\{A^n; n = 1, 2, \dots\}$. Furthermore, $B(H)/M$ is subdirectly irreducible with idempotent heart.*

Proof. M exists by Zorn's lemma since the ideal of finite rank operators misses $\{A^n\}$. If $M \subsetneq K$, an ideal of $B(H)$, then there is an n so that $A^n \notin K$. Hence, by the theorem above, $\langle A \rangle = \langle A \rangle^n = \langle A^n \rangle \subset K$. Therefore $A \in \bigcap \{K; K \supsetneq M\}$, so $B(H)/M$ is subdirectly irreducible. Also, since M is automatically prime, the heart G/M is idempotent.

The example now follows easily. If C denotes the compact operators, and F the operators of finite rank, it follows that C/F is Jacobson radical since, by a theorem of Kaplansky [1], C is the only non-zero primitive ideal of $B(H)$. Therefore, $(C/F)/(M/F) \cong C/M$ is Jacobson radical. Also, C/M is subdirectly