

# *Degenerate Evolution Equations and Applications*

R. E. SHOWALTER

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**1. Introduction.** We shall consider the Cauchy problem for the equation  
(1.1) 
$$A(t)u(t) + D_t B(t)u(t) = f(t)$$

in which  $\{A(t)\}$  and  $\{B(t)\}$  are bounded and measurable families of linear operators between Hilbert spaces. In Section 2 we show that the problem is well-posed when the  $\{B(t)\}$  are non-negative,  $\{A(t) + \lambda B(t)\}$  are coercive, and the operators depend smoothly on  $t$ . The equation is degenerate since the coefficient operators are not necessarily invertible and may in fact vanish. Sections 3 and 4 contain some applications to various boundary value problems. The examples are chosen simply to indicate the large class of problems to which the abstract results can be applied, and so we do not attempt to present the most technically refined results in any sense.

The abstract problem which we shall formulate and resolve in Section 2 was initially motivated by boundary value problems for partial differential equations of the form

$$(1.2) \quad \frac{\partial}{\partial t} \{b_0(x, t)u(x, t) - b(x, t)\Delta u(x, t)\} - \Delta u(x, t) = f(x, t).$$

Such equations appear in various applications [1, 8, 9, 16] with non-negative (real) coefficients determined by material constants. The coefficient  $b(x, t)$  usually has the dimensions of viscosity and distinguishes (1.2) from the standard equation of heat conduction. This equation is elliptic in that region where  $b_0(x, t) = b(x, t) = 0$ , parabolic where  $b(x, t) = 0$  and  $b_0(x, t) > 0$ , and of Sobolev type where  $b(x, t) > 0$ . Since we permit our abstract equation (1.1) to degenerate, the coefficients in our model (1.2) may vanish identically on certain space-time regions. It became apparent that the abstract results obtained below would also give existence and uniqueness of weak solutions of boundary value problems of much more general type than was first anticipated, so we have attempted to illustrate some of these diverse applications in the examples.