

Continuous Maps with 0-dimensional Branch Set

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Communicated by A. H. WALLACE

1.1. Theorem. Let M^{p+2} and N^p be (topological) manifolds with $p \geq 2$, let $f : M^{p+2} \rightarrow N^p$ be a (continuous) map, and let B_f be the set of points in M^{p+2} at which f fails to be locally topologically equivalent to the projection map $\rho : \mathbf{R}^{p+2} \rightarrow \mathbf{R}^p$. Assume that $\dim B_f = \dim f(B_f) \leq 0$, and, if $p \geq 3$, that $f(B_f)$ is locally tame in N^p . Furthermore, assume that for each $x \in M^{p+2}$ the component $\Gamma(x)$ of $f^{-1}(f(x))$ containing x is not merely $\{x\}$. Then B_f is discrete.

In a sequel [4] we show that: (1) the hypotheses on $\Gamma(x)$ may be omitted; (2) if $B_f \neq \emptyset$, then $p = 2$; and (3) if f is a C^{p+2} map, the tameness hypotheses can be omitted. For a C^1 map, B_f is contained in the critical set X by the Rank Theorem [3, (1.6)], and the hypothesis on B_f may thus be replaced by the same one on X in the C^1 case.

This paper is a sequel to [3], and a discussion of related results, questions, and examples is given there, especially in [3, (1.7)]. Examples of maps satisfying the hypotheses of (1.1) are complex polynomial maps with isolated critical point at 0, as discussed in [10] (e.g. $\sigma(z, w) = z^j + w^k$, $j > k \geq 2$).

1.2. Conventions. In order to read the proofs of this paper, the reader will need to have [3] at hand, and we assume its conventions and definitions. The symbol H^q (resp. H_q , H_c^q) refers to singular homology (resp., Alexander cohomology, Alexander cohomology with compact supports), \mathbf{Z} is the group of integers, and \mathbf{Z}_2 the field of order 2. When coefficients are not specified, \mathbf{Z}_2 is used, and $d^q(X) = \dim(H^q(X))$. A map is a continuous function.

2. Preliminaries.

2.1. Definition. Given M^n and N^p manifolds with (possibly empty) boundary, $n \geq p$, and a map $f : M^n \rightarrow N^p$, we define the *branch set* $B_f \subset M^n$. Let $\mathbf{R}^m_+ = \{x \in \mathbf{R}^m : x_m \geq 0\}$, let $F = \mathbf{R}^{n-p}$ or \mathbf{R}^{n-p}_+ , and let $G = \mathbf{R}^p$ or \mathbf{R}^p_+ (not respectively). Then $x \notin B_f$ if and only if f at x is locally topologically equivalent to the projection map $\pi : F \times G \rightarrow G$ at $(0, 0)$. Occasionally the notation $B(f)$ is used.