

# Special Cohomology and Branched Coverings

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**1. Introduction.** A branched covering is a light open map  $f: M \rightarrow N$  for which there exists a set  $B_f$  (the branch set) such that  $f|_{(M - B_f)}$  is a covering projection onto  $(N - fB_f)$ . If  $f|_{f^{-1}fB_f}$  is a homeomorphism onto  $fB_f$ , the map is called a Montgomery–Samelson (M–S) covering. We will consider only the case in which  $M$  and  $N$  are compact connected  $n$ -manifolds. In this case  $\dim B_f \leq n - 2$  [1] and  $f|_{(M - B_f)}$  is a finite-to-one map. A special homology theory has recently been used to study M–S coverings [4, 7, 8, 9]. The construction in [7] which is the cornerstone of this work is not valid in the generality there claimed. It is valid for orbit maps of semifree finite group actions with tamely embedded branch set images. Such maps are called tame *regular M–S coverings* and constitute an important class of branched coverings. In this paper we construct a special homology and cohomology for regular M–S coverings. We specialize the F  ry spectral sequence to this situation and make a few applications. We investigate M–S coverings on spheres, lens spaces and some spheres with handles. We also consider branched coverings on  $S^3$  and some flat manifolds.

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**2. The major tools.** Throughout this paper, *degree* means geometric degree [2]. We will use degree to mean integral degree and absolute degree whenever they coincide with geometric degree without explicit mention of this fact.

**Theorem 1.** *Let  $G$  be a finite group acting semi-freely on the  $n$ -manifold  $M$ . Let the orbit space  $N$  be a manifold and let  $f: M \rightarrow N$  denote the orbit map and  $B_f$  its branch set. If  $fB_f$  is tamely embedded and not empty, then  $G$  is  $\mathbf{Z}_d$ . Therefore, Smith theory implies that for each prime  $p$  dividing  $d$  there exist graded vector spaces  $H_*^{\rho}(M)$ ,  $H_*^{\rho}(M, B_f)$  ( $\rho = \sigma, \tau$ ) such that*

- (1)  $H_*^{\sigma}(M) = H_*^{\sigma}(M, B_f) \oplus H_*(B_f)$
- (2)  $H_*^{\tau}(M, B_f) = H_*(N, fB_f)$