

Growth Conditions, Spectral Operators and Reductive Operators

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Introduction. Throughout this paper T will denote a bounded linear operator defined in a Hilbert space H .

N. Dunford [4, page 1948] has proved the following theorem.

Theorem. *Let T be a bounded spectral operator, let E be its resolution of the identity, and T_δ its restriction to the subspace $E(\delta)H$. Then T is of type $m - 1$ if and only if there is a constant K independent of the Borel set δ such that*

$$(*) \quad \|(z - T_\delta)^{-1}E(\delta)\| \leq K/\text{dist}(z, \delta)^m, \quad z \notin \delta, \quad |z| \leq \|T\| + 1.$$

The proof of this theorem uses the fact that the above condition (*) can be replaced by the following condition:

$$(**) \quad \|(T - z)^m E(\delta)\| \leq M |\delta|, \quad \delta \text{ Borel},$$

where $|\delta|$ denotes the diameter of δ , $z \in \delta$, and M is a constant independent of δ . In this paper we will show that the above theorem remains true under a much weaker form of the condition (**); more precisely we will prove that a spectral operator T is of type $m - 1$ if and only if at each point $z \in \sigma(T)$ there exists a decreasing sequence δ_n of closed rectangles containing z as an interior point and converging to $\{z\}$ such that

$$\lim_{n \rightarrow \infty} \|(T - z)^m E(\delta_n)\| = 0.$$

(See Corollary 2.2 and Proposition 1.3.) This result gives us, in some sense, a local method to approach the problem of finding the type of a spectral operator; and in particular it enables us to generalize a result of J. G. Stampfli to the case where $\sigma(T)$ is nowhere dense (see Theorem 2.3) and a criterion of N. Dunford to the case where the spectrum lies on a C^1 Jordan curve (see Theorem 2.5).

As another application we will show that a reductive operator whose resolvent satisfies certain growth conditions is necessarily normal. This is a continuation of the work done in [6] and is discussed in Section 3.