

Some Third Order Initial Value Problems Arising in Magnetohydrodynamics

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Communicated by the Editors

1. Introduction. In this article, we discuss the asymptotic behavior of two third order nonlinear initial value problems that arise in an investigation of a nonlinear, resistive hydromagnetic boundary layer in an inviscid rotating fluid.

A complete derivation of the third order nonlinear differential equation and the initial conditions is presented in Section 2. There, the Navier-Stokes equations, which are coupled with Maxwell's equation to describe the fluid flow and the magnetic fields, are reduced to the initial value problems

$$s''' + 3ss'' + \frac{1}{2}(s')^2 + 2s^2s' = 0, \quad ' = d/dt$$
$$s(0) = s''(0) = 0, \quad s'(0) = \pm 2.$$

The purpose of this paper is to study these and related problems.

The outline of the remainder of the paper and the nature of our results are now indicated. Section 3 establishes that the solution of the initial value problem with a positive initial slope is bounded and monotone increasing. It, therefore, approaches a positive constant as t approaches infinity and, as such, demonstrates a boundary layer character. In Section 4, the initial value problem with a negative initial slope is considered. It is shown that the solution of this initial value problem has one zero on $(0, \infty)$; then, after this zero, the solution is positive and monotone increasing. The solution is bounded and, hence, it also exhibits a boundary layer character. Numerical solutions of the above initial value problems are presented in Section 2 following the derivation of the third order equation. These computations were helpful guides for developing the analysis given in the paper.

In Section 5 we investigate the asymptotic behavior of the solutions of a class of initial value problems associated with the third order differential equation. In general, it is shown that the solutions of initial value problems in this class have a behavior similar to the two physically important problems mentioned above; that is, as t approaches infinity, they approach constants.