

Rotation Invariant Measure Algebras on Euclidean Space

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§1. Introduction. In this paper we study the convolution and maximal ideal structure of the Banach algebra of rotation invariant finite measures on \mathbf{R}^n , $n \geq 2$ and of several related subalgebras of the full measure algebra of \mathbf{R}^n . Our first major result (2.5) shows that the convolution of two continuous rotation invariant measures is absolutely continuous (with respect to Lebesgue measure). This is parallel to a result of Dunkl [1] on zonal measures on spheres. Several facts about rotation invariant measures follow from this algebraic result. In (2.7) we show that such a measure is continuous if and only if its Fourier-Stieltjes transform vanishes at infinity. In section 3 we are able to determine the maximal ideal space or spectrum of the algebra of all such measures. Specifically, (3.3) shows this spectrum is homeomorphic to $[0, \infty]$ which should be viewed as the one-point compactification of the spectrum of the algebra of rotation invariant L_1 -functions (see [8]). Moreover, each homomorphism of the rotation invariant measure algebra is given by integration against a bounded Borel function (3.3) and (3.4).

In section 6 we generalize our work to show that if K is any compact *subgroup* of orthogonal transformations of \mathbf{R}^n , $n \geq 2$, whose identity component acts irreducibly, then most of the results mentioned above extend to the algebra of K -invariant measures on \mathbf{R}^n . As a prelude to this section we find it useful to derive some general Banach algebra theorems in section 4 which show (4.3) that the spectrum of the algebra of K -invariant L_1 -functions on \mathbf{R}^n is just the space \mathbf{R}^n/K of K -orbits in \mathbf{R}^n . This generalizes work of Reiter [8] on the rotation invariant case.

In section 5, as a further prelude to section 6, we prove some general results on absolute continuity of convolutions in \mathbf{R}^n which supplement (2.5). One such result (5.2) shows that the convolution square of the surface measure on a compact analytic hypersurface is absolutely continuous.

The paper concludes with an appendix which contains several more analytic proofs of the basic results of section 2. These proofs do not easily generalize