

Perturbations of C^ -algebras*

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Introduction. In their recent paper, [3], R. V. Kadison and D. Kastler initiated the study of perturbations of von Neumann algebras. The ultimate aim of this study is to show that “neighbouring” (in the sense that their unit balls are norm close) von Neumann algebras are unitarily equivalent. A partial result in this direction is contained in [4]. Since their definition of the distance between operator algebras makes perfectly good sense for C^* -algebras (in fact, for normed linear spaces) it seems that the natural place to investigate the problem of perturbations of operator algebras is in the context of norm closed operator algebras, that is, C^* -algebras. This paper may be viewed as a first step in that direction.

In section 1 we show that neighbouring C^* -algebras have isomorphic lattices of ideals and hence, have homeomorphic primitive ideal spaces. In section 2 we show that if \mathfrak{A} and \mathfrak{B} are “close” then \mathfrak{A} is liminal (respectively, postliminal, antiliminal) if and only if \mathfrak{B} is liminal (respectively, postliminal, antiliminal). It is also shown in section 2 that neighbouring C^* -algebras have neighbouring “composition series” (see section 2 for the definition). Finally in section 3 we show that neighbouring C^* -algebras have homeomorphic spectra.

Notations and definitions. All ideals mentioned in this paper are assumed to be closed and two-sided. If \mathfrak{A} is a set of bounded linear operators on the Hilbert space \mathfrak{H} , then we use \mathfrak{A}' to denote the set of all bounded linear operators on \mathfrak{H} which commute with every element of \mathfrak{A} . We use the notation \mathfrak{A}_1 to denote the set of all operators in \mathfrak{A} whose bound is less than or equal to 1. The algebra of all bounded operators on \mathfrak{H} is denoted by $\mathfrak{B}(\mathfrak{H})$. If \mathfrak{A} is an algebra of operators on \mathfrak{H} with identity, then the identity of \mathfrak{A} is denoted by $1_{\mathfrak{A}}$. However, the identity operator on \mathfrak{H} will be denoted by 1. For each subset \mathfrak{F} of $\mathfrak{B}(\mathfrak{H})$ and each bounded operator A on \mathfrak{H} we let

$$\|A - \mathfrak{F}\| = \inf \{\|A - F\| : F \text{ in } \mathfrak{F}\}$$

Definition. If \mathfrak{A} and \mathfrak{B} are linear subspaces of $\mathfrak{B}(\mathfrak{H})$,

$$\|\mathfrak{A} - \mathfrak{B}\| = \sup \{\|A - \mathfrak{B}_1\|, \|\mathfrak{B} - \mathfrak{A}_1\| : A \text{ in } \mathfrak{A}_1, B \text{ in } \mathfrak{B}_1\}.$$