

Semihandles and 4-Manifolds

DALLAS E. WEBSTER

Communicated by the Editors

1. Introduction. In [3] the concepts of semihandles and semistraightening were introduced and results were obtained and applied to ordinary handles. Some of these results had been known previously in a different format but could be restated in terms of semihandles. Distinguishing these semihandles with a name immediately inspires indulgence in the luxury of formulating and solving problems about them. Certain questions, which somehow were previously obscured, now seem to arise quite naturally. In this paper we present a couple of related semihandle conjectures which are motivated by analogous results for handles. The conjectures will become increasingly interesting as we show that they would solve several of the most significant outstanding problems of 4-manifolds, even though the conjectures themselves are 5-dimensional in nature. Results of a similar, though more technical, nature have been announced by Kirby [6].

2. Preliminaries. Let \mathbf{R}^n denote Euclidean n -space with the sum-of-the-absolute-values norm and rB^n the n -cell in \mathbf{R}^n of radius r centered at 0. Let $B^n = 1B^n$ and $J^{n-1} = \{1\} \times B^{n-1} \cup B^1 \times \partial B^{n-1} \subseteq B^1 \times B^{n-1} = B^n$. A (k, n) -handle (respectively, a (k, n) -semihandle) is a TOP homeomorphism $h: B^k \times \mathbf{R}^n \rightarrow V$ where V is a PL $(k+n)$ -manifold and h is PL on $\partial B^k \times \mathbf{R}^n$ (respectively, $J^{k-1} \times \mathbf{R}^n$). A (k, n) -(semi-) handle $h: B^k \times \mathbf{R}^n \rightarrow V$ can be (semi-) straightened if and only if there is an isotopy $h_t: B^k \times \mathbf{R}^n \rightarrow V$ ($t \in [0, 1]$) such that $h_0 = h$, h_1 is PL on $B^k \times B^n$, and $h_t = h$ on $\partial B^k \times \mathbf{R}^n$ ($J^{k-1} \times \mathbf{R}^n$) and outside a compact set.

We wish to investigate the following two classes of propositions.

$H(k, n)$: Every (k, n) -handle can be straightened.

$S(k, n)$: Every (k, n) -semihandle can be semistraightened.

Quite a bit is already known about these propositions. A portion of this knowledge is summed by the following statements.

1. $H(k, n)$ is true for $k + n \geq 5$ and $k \neq 3$. (See [7] and [4].)
2. $H(3, n)$ is false for $n \geq 2$. (See [4].)