

# Differentiable Maps with 0-Dimensional Critical Set, II

P. T. CHURCH & J. G. TIMOURIAN

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## 1. Introduction.

**1.1. Theorem.** Let  $f: M^n \rightarrow N^p$  be a  $C^n$  map with  $n - p = 0, 1$  or  $2$  and  $p \geq 2$ , let  $R_{p-1}(f)$  be its critical set, and let  $\dim R_{p-1}(f) = \dim f(R_{p-1}(f)) \leq 0$ . Then, at each  $x \in M^n$ ,  $f$  is locally topologically equivalent to one of the following maps:

- (a) the projection map  $\rho: \mathbf{R}^n \rightarrow \mathbf{R}^p$ ,
- (b)  $\sigma: \mathbf{C} \rightarrow \mathbf{C}$  defined by  $\sigma(z) = z^d$  ( $d = 2, 3, \dots$ ), where  $\mathbf{C}$  is the complex plane,
- (c)  $\tau: \mathbf{C} \times \mathbf{C} \rightarrow \mathbf{C} \times \mathbf{R}$  defined by  $\tau(z, w) = (2z \cdot \bar{w}, |w|^2 - |z|^2)$ , where  $\bar{w}$  is the complex conjugate of  $w$ , or
- (d) a  $C^n$  map  $\eta: \mathbf{R}^4 \rightarrow \mathbf{R}^2$  which is locally topologically equivalent to  $\rho$  except at one point.

In particular, either  $f$  is locally topologically equivalent to  $\rho$  at every point, or  $(n, p) = (2, 2), (4, 3)$ , or  $(4, 2)$ . Examples of maps  $\eta$  are the complex polynomial maps defined by  $\eta(z, w) = z^j + w^k, j > k \geq 2$  [9, p. 3]. The case  $n - p = 0$  or  $1$  is given in [4], and only  $n - p = 2$  is considered here.

For background information see the introduction of [4]. In particular (1.1) answers a fortiori for  $n - p \leq 2$  a question of Milnor [9, p. 100] (see [4, (1.7)]).

**1.2. Conventions.** A symbol such as  $M^n$  denotes a separable  $n$ -manifold, without boundary unless otherwise specified. A manifold with boundary may have an empty boundary. Homology is singular, cohomology is Alexander, coefficients are  $\mathbf{Z}_2$ , and  $d^q(X) = \dim(H^q(X))$ . The branch set  $B_f \subset M^n$  is the set of points at which  $f$  fails to be locally topologically equivalent to projection ([4, (1.5)] or [5, (2.1)]). For detailed conventions and definitions see [4].

**1.3. Outline.** Let  $f: M^{p+2} \rightarrow N^p$  ( $p \geq 2$ ) be a map with  $\dim B_f = \dim f(B_f) \leq 0$  and  $f(B_f)$  locally tame. Under an additional assumption it was shown in [5] that  $B_f$  is discrete. In §2, starting with this result, or more precisely with