

# *Exponential Decay of Solutions to Hyperbolic Equations in Bounded Domains*

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**§1. Introduction.** In this paper we consider the decay of solutions to hyperbolic equations in bounded domains. In contrast to equations in exterior domains where local decay takes place because signals radiate to infinity, in the bounded case there must be some direct dissipative mechanism. We discuss two types. In one kind the energy of a wave will decrease when it passes through some fixed subregion of the domain. The second sort of decay occurs at the boundary; energy is lost when a wave is reflected from a fixed subset of the boundary.

Iwasaki [4, §2.2] has shown under broad hypotheses that either sort of decay mechanism forces the energy of solutions to decrease to zero as  $t \rightarrow +\infty$ . Here, we investigate conditions which imply that the decay is exponential.

For all problems the basic hypothesis is that there is a time  $T > 0$  such that a bicharacteristic ray starting at any point and suitably reflected at the boundary reaches the region where decay takes place in time less than  $T$ . The idea behind this assumption is that the energy of a solution to a hyperbolic equation  $Lu = 0$  is largely carried along bicharacteristic rays of  $L$ , so such a hypothesis says that all such rays spend plenty of time in the region where energy loss occurs.

We treat problems on manifolds without boundary in section 2 and 3. In section 4 we study problems with boundary, but we can only treat the case of one space variable.

**§2. First order systems on a manifold without boundary.** Let  $L = \partial/\partial t - G(x, D_x)$ , where  $G$  is a first order system of differential operators with smooth coefficients on  $M$ . We assume  $M$  is a smooth compact manifold without boundary endowed with a volume element. Assume  $G$  satisfies the dissipative condition  $G + G^* = -F(x)$  where  $F$  is a non-negative self-adjoint matrix function.  $L$  is called a dissipative symmetric hyperbolic system.