

Factorization of Positive Operator Valued Functions on a Banach Space

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1. Introduction. The main purpose of this note is to extend the well-known results of R. G. Douglas [4] on factoring positive operator valued functions on a Hilbert space to positive operator valued functions on a Banach space. The problem of factoring positive operators on a Hilbert space plays an important role in the study of Hilbert space valued stationary processes (see, for example, [8], [9], [10]). There is a similar tie between the factorability of positive operator valued functions on a Banach space and Banach space valued stationary processes [1], [2], [3], [12].

When f is a positive operator valued function on a Hilbert space, $f^{1/2}$, the square root of f whose existence is known, plays a significant role. But when f is a positive operator valued function on a Banach space \mathfrak{X} the existence of a square root in the ordinary way does not make sense. Nevertheless we can prove (*cf.* Lemma 3.1) the existence of a measurable operator valued function A on \mathfrak{X} into some auxiliary Hilbert space which behaves almost like a square root in the sense that $f = A^*A$. The operator valued function A , called a quasi square root, enables us to extend to the Banach space case a lemma of Helson ([6], p. 117) and the main lemma of Douglas [4].

In §2 we set up necessary terminologies and state some known results. Section 3 includes the proof of existence of a quasi square root and two lemmas on the characterization of factorability of a positive operator on a Banach space. The results of §4 extend in a natural way most of the work of R. G. Douglas [4] to the Banach space case. In establishing these results we make use of our fundamental lemmas proved in §3 and Douglas' techniques employed in [4].

§2. Ancillary results. In this paper all Banach spaces and Hilbert spaces are assumed to be separable. Let \mathfrak{K} be a Hilbert space and let $L_2(\mathfrak{K})$ denote the Hilbert space of functions defined on the unit circle with values in \mathfrak{K} that are weakly measurable and have square summable norm with respect to (w.r.t.) Lebesgue measure. The $L_2(\mathfrak{K})$ inner product of two functions $g_1 = g_1(e^{i\theta})$ and