

Asymptotic Behavior and the Cauchy Problem for Ultrahyperbolic Operators

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1. Introduction. We consider the operator $L = -A_x + B_y$ where A_x and B_y are second order linear elliptic operators in $x = (x_1, \dots, x_m)$ and $y = (y_1, \dots, y_n)$, respectively. If both $m \geq 2, n \geq 2$ the operator L is ultrahyperbolic; while if either m or n is 1, L is hyperbolic. We shall be concerned with equations of the form

$$(1.1) \quad Lu = F(x, y, u, \nabla_x u, \nabla_y u)$$

in which F satisfies for all values of the arguments an inequality of the form

$$(1.2) \quad |F(x, y, z, a, b)| \leq \varphi_0(y) |z| + \varphi_1(y) |a| + \varphi_2(y) |b|.$$

When (1.2) holds the methods we employ yield information for the differential inequality

$$(1.3) \quad |Lu| \leq \varphi_0(y) |u| + \varphi_1(y) |\nabla_x u| + \varphi_2(y) |\nabla_y u|.$$

The asymptotic behavior of solutions of differential inequalities has been studied extensively. For operators in which B_y is of the first order and $n = 1$, the most comprehensive results are to be found in Agmon and Nirenberg [1]. If B_y is of second order and $n = 1$, lower bounds and estimates for asymptotic decay were obtained by Levine [2], Ogawa [5], and the authors [4, 7].

In Sections 2 and 3, we consider solutions $u = u(x, y)$ of (1.3) in a region $D \times \Gamma$ where D is a bounded domain in x -space and Γ is a positive cone in y -space, *i.e.*, closed under multiplication by positive scalars. We study the behavior as $|y| \rightarrow \infty$ of solutions which vanish on the boundary of $D \times \Gamma$. In the simplest case with A_x and B_y Laplace operators, $F = 0$, and $m = n$ it is clear that

$$(1.4) \quad u(x, y) = \sin kx_1 \sin kx_2 \cdots \sin kx_n \sin ky_1 \sin ky_2 \cdots \sin ky_n$$

is a solution of $\Delta_x u - \Delta_y u = 0$ which vanishes on the boundary of the domain D consisting of an n -dimensional cell of side π . The solution also vanishes on $D \times \partial\Gamma$. We observe that (1.4) is periodic and hence does not decay at all as $|y| \rightarrow \infty$.